Experimental Investigation of Turbulent Momentum Transfer in a Neutral Boundary Layer over a Rough Surface
S. Tomas, O. Eiff, V. Masson

To cite this version:

HAL Id: hal-00657250
https://hal.archives-ouvertes.fr/hal-00657250
Submitted on 6 Jan 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Experimental investigation of the turbulent momentum transfers in a neutral boundary layer over a rough surface

SEVERINE TOMAS¹, OLIVIER EIFF²,³ and VALERY MASSON⁴

Abstract. The turbulent characteristics of the neutral boundary layer developing over rough surfaces are not well predicted with operational weather-forecasting models. The problem is attributed to inadequate mixing-length models, to the anisotropy of the flow and to a lack of controlled experimental data against which to validate numerical studies. Therefore, in order to address directly the modelling difficulties of the development of a neutral boundary layer over rough surfaces and to investigate the turbulent momentum transfers of such a layer, a set of hydraulic flume experiments were carried out. In the experiments, the mean and turbulent quantities were measured by a particle image velocimetry (PIV) technique. The measured velocity variances and fluxes ($\overline{u_i'u_j'}$) in longitudinal vertical planes allowed the vertical and longitudinal gradients ($\partial/\partial z$ and $\partial/\partial x$) of the mean and turbulent quantities (fluxes, variances and third-order moments) to be evaluated and the terms of the evolution equations for $\partial e/\partial t$, $\partial \overline{u'^2}/\partial t$, $\partial \overline{w'^2}/\partial t$ and $\partial \overline{u'w'}/\partial t$ to be quantified, where $e$ is the turbulent kinetic energy. The results show that the pressure-correlation terms allow the turbulent energy to be transferred equitably from $\overline{u'^2}$ to $\overline{w'^2}$. It appears that the repartition between the constitutive terms of the budget of $e$, $\overline{u'^2}$, $\overline{w'^2}$ and $\overline{u'w'}$ is not significantly affected by the development of the rough neutral boundary layer. For the whole evolution, the transfers of energy are governed by the same terms which are also very similar to the smooth-wall case. The PIV measurements also allowed the spatial integral scales to be computed directly and to be compared with the dissipative and mixing lengths scales, which were also computed from the data.

Keywords: Functional lengths, Integral scales, Neutral boundary layer, Particle image velocimetry measurements, Rough walls, Turbulent budgets.

¹S. Tomas
National Center for Atmospheric Research/GAME Météo-France, CNRS, France
current affiliation: Cemagref, UMR G-EAU, LERMI, France
severine.tomas@cemagref.fr

²O. Eiff
University of Toulouse, INPT, UPS, IMFT (Institut de Mécanique des Fluides de Toulouse), France

³O. Eiff
CNRS, IMFT, France

⁴V. Masson
National Center for Atmospheric Research/GAME Météo-France, CNRS, France
1 Introduction

Roughness effects on turbulent flow over surfaces have been studied since the mid-19th century. For atmospheric boundary layers, which are usually rough, roughness effects have also received particular attention; see for example the review of Raupach et al. (1991). When the roughness elements are high enough, the offset constant in the smooth-wall logarithmic law changes, as discussed in most textbooks (e.g., Schlichting and Gersten 1979). The roughness effects are accounted for in the logarithmic law via differently defined but equivalent roughness parameters: (i) the roughness length, used mostly in the atmospheric context, (ii) the equivalent sand roughness, or finally (iii) the roughness function (see Jiménez 2004). Yet, an important and long-standing question is whether the outer-layer flow \((z^+=u^+z/\nu > 50)\), where \(u^+\) is the friction velocity and \(\nu\) the kinematic viscosity (e.g. Pope 2000) depends on the wall roughness. Some studies, investigating the effect of roughness on the turbulence structure (Antonia and Luxton 1971; Bandyopadhyay 1987; Raupach et al. 1991), conclude that there is an outer-layer similarity over smooth and rough walls. In this context, Townsend’s (1976) similarity hypothesis is often invoked as an implicit “wall-similarity” (Raupach et al. 1991) deduced from the general Reynolds number similarity (Townsend 1976). The recent experimental studies of Flack et al. (2005) and Schultz and Flack (2007) support the outer-layer similarity of smooth- and rough-wall boundary layers in terms of both the mean flow and the Reynolds stresses. Similarly, the combined experimental and numerical investigation of Krogstad et al. (2005) suggests an outer-layer very little affected by the roughness elements. However, an outer-layer structure unaffected by the wall condition is not always found. For example, the experimental study of Krogstad et al. (1992) showed that the roughness at the wall influences the mean velocity and the turbulent stresses into the outer-layer.

In a recent review, Jiménez (2004) readdressed this issue. He stressed that in addition to the roughness Reynolds number usually considered, i.e. \(Re_s = k_s u^+\nu\), where \(k_s\) is the equivalent sand roughness, care needs to be taken to account for the fact that the roughness sublayer will increasingly affect a significant portion of the logarithmic zone for low blockage ratios, \(\delta/z_h\), where \(\delta\) is the boundary-layer height and \(z_h\) the height of the rough elements. Jiménez (2004) estimated that to be free from direct roughness effects, \(\delta/z_h\) must be greater than at least 40. Thus, in order to attain fully rough regimes, the roughness Reynolds number needs to be greater than about 80. He concludes that the Reynolds number \(Re_{\delta^+} = \delta u^+/\nu\) (also called Kármán number \(\delta^+\)) should be greater than 4000. These are conditions that are usually satisfied in real atmospheric boundary-layer flows, with the possible exception of urban boundary layers.
Owing to the development of remote sensing technologies, in-situ measurements have become accurate enough to investigate the turbulence of the atmospheric boundary layer (e.g. Poulos et al. 2002; Drobinski et al. 2007 and Kunkel and Marusic 2006). The experiments of Kunkel and Marusic (2006) over the western Utah Great Salt Lake Desert covered very high Kármán numbers ($\delta^+ \approx 10^6$) with very small relative roughness heights ($k_s/\delta \approx 10^{-4}$). The results show good agreement with the similarity formulations (Marusic and Kunkel 2003) based on Townsend’s attached eddy model (1976) and thus support Townsend’s outer-layer similarity. As the authors suggest, some of the observed outer-layer differences in comparison with other studies might be attributed to blockage effects and/or low Reynolds numbers.

Yet, most boundary-layer studies, especially those concerned with roughness effects, only consider the fully developed boundary layer at equilibrium. The development of boundary layers is usually only considered in the context of a transition of well-developed boundary layer subject to a sudden change of wall roughness, also referred to as internal boundary layer, recent examples including those of Cheng and Castro (2002a; 2002b) and Belcher et al. (2003). One exception is the study of Castro (2007) who examined boundary layers developing in wind tunnels over various roughnesses at relatively high Reynolds numbers. However, he focused only on the mean flow for which he concluded that outer-layer flow similarity holds even for surprisingly low blockage ratios, down to $\delta/z_h > 5$, (i.e. significantly smaller than the criterion suggested by Jiménez (2004)). He did not examine the turbulence structure nor the functional and integral length scales.

A better knowledge of the turbulence structure including the functional and integral length scales is necessary to validate and improve numerical simulations. This includes 1D boundary-layer prediction models which are computationally inexpensive but need to be improved. For example, some 1D models still use simple expressions for the mixing length in neutral boundary layers, such as the Blackadar mixing length (Blackadar 1962)

$$ l = \kappa z l_0/(\kappa z + l_0) $$

where $l_0$ is an asymptotic value of the order of a few hundred metres which are not appropriate. This mixing length only depends on height, and not on the flow (it is equal to $\kappa z$ near the ground and to $l_0$ high above). The large values in the free troposphere predicted by this model are not valid, however, because the stable stratification tends to strongly limit the size of the eddies. Some mixing lengths tend to correct this with stability modulating the mixing length (Bougeault and Lacarrere, 1989), as in the mesoscale meteorological model Meso-NH (Lafore et al., 1998) or the operational model AROME (Bouttier, 2003; Ducroq et al., 2005; Bouttier, 2007; Seity et al., 2010). Unfortunately, such mixing length parameterization does not correctly take into account the neutral case. It is not defined
if there is no thermal inversion above the boundary layer. Indeed, at the top of the atmospheric boundary layer, there is usually a zone of high stability which separates the boundary layer from the free troposphere above. This is the reason why the mixing lengths are often based on a criterion of stability: in stable conditions, vortices are locally inhibited and mixing lengths are respectively small. In neutral or in unstable conditions, the mixing lengths are relatively large based on the assumption that vortices reach the boundary-layer top as defined by the area of high stability. While this assumption is true in unstable conditions, it is more questionable in neutral conditions where the turbulent structure is only determined by the surface friction and not related to the temperature gradient. Consequently, in a neutral boundary layer, the model overestimates the mixing length. Thus, since no current formulation of the mixing length takes into account the specific features of the neutral boundary layer and its streamwise vortices, except near the surface (Redelsperger et al. 2001; Drobinski et al. 2007), there appears to be no formulation which satisfactorily predicts the complete structure of neutral boundary layers. This is, in part, due to the lack of data under known and controlled conditions. Hess and Garratt (2002) used in-situ observations to evaluate the first-order models of Lettau (1962) and Blackadar (1962 and 1965) and the higher-order models of Freeman and Jacobson (2002) and Xu and Taylor (1997a; b) in the neutral case. While the first-order models generally performed well, by fitting a free parameter, the second-order models performed poorly. Hess and Garratt (2002) attribute this behaviour to deviations from the idealized conditions the higher-order models are based upon. In summary, it can be concluded that there is a need for better experimental data verification under controlled conditions, with access to the turbulent structure of the boundary layers.

For this purpose, it is proposed here to experimentally investigate the development of a neutral boundary layer under a potential free-stream flow over a rough surface. The study aims to provide an idealized and simple case for analysis and validation, for which no data-set under controlled and known conditions seems to be available. In order to attain high enough Reynolds numbers while retaining small enough relative roughness heights (Jiménez 2004), the flow was simulated in the large water flume of the French National Center of Meteorological Research Center in Toulouse. Vertical two-dimensional velocity fields, spanning the development of the boundary layer, were measured via a particle image velocimetry (PIV) technique. The resulting experimental data describe the inertial and outer-layer, i.e. above the surface layer, of a fully rough and developing neutral boundary layer, in terms of the mean and turbulent quantities under known conditions with a high level of accuracy. The measured velocity variances and fluxes ($u'^2$, $w'^2$ and $u'w'$) in a longitudinal vertical plane allow the vertical and longitudinal gradients ($\partial/\partial z$ and $\partial/\partial x$) of the mean and turbulent quantities.
(fluxes, variances and third-order moments) to be evaluated. They also enable the advection, the dynamical production, the turbulent transport, the dissipation and pressure-correlation terms of the $\partial e/\partial t$, $\partial u'^2/\partial t$, $\partial w'^2/\partial t$ and $\partial u'w'/\partial t$ evolution equations to be estimated. Spatial correlation analysis permits the longitudinal and vertical integral lengths to be estimated and to be compared to dissipation lengths and mixing lengths evaluated directly from the data. The intent is to improve the parameterizations of functional lengths when equilibrium is reached. The data should also help assess LES simulations or other closure models simulations of developing atmospheric-type rough wall neutral boundary layers.

The experimental procedure is presented in Sect. 2 and the boundary-layer development is discussed in Sect. 3. Then, the study focuses on the turbulent energy budgets and the transfers between the Reynolds stresses. The spatial integral scales as well as the dissipative and mixing lengths are discussed and compared in Sect. 5. The main conclusions are summarized and discussed in Sect. 6.

2 Experimental procedure

2.1 Boundary-layer flow facility

The experiments were carried out in a large, wide and high hydraulic flume (22 m long, 3 m wide and 1 m high) (Fig. 1a) to obtain high Reynolds numbers, to generate a two dimensional flow and to minimize other boundary effects. The flume was equipped with a series of grids to reduce the incident turbulence intensity and homogenize the incident flow, such that it can be considered uniform and potential. A one-metre long smooth surface behind the last grid was installed to allow the uniform and potential flow with a small smooth and laminar boundary layer to approach the roughness elements. A turbulent and rough boundary layer was then allowed to develop over a 12.3 m long and 3 m wide horizontal rough surface. The temperature of the water was maintained within $\pm 0.2$ K, assuring neutral flow.

A Laser-Doppler-Velocimetry (LDV) technique was used for an initial estimation of the rough-wall boundary-layer thickness and for the validation of the flume characteristics. Vertical profiles of the mean velocity and turbulent intensity were obtained at several abscissas ($x = [-0.16; 0.1; 6; 10.7; 12.4]$ m) for three transversal $y$-positions ($y = [-0.5; 0; 0.5]$ m). For the incident free-stream velocity chosen ($U_{\infty,0} = 0.35$ m s$^{-1}$), the turbulent intensity and the velocity profiles for the three $y$-positions are essentially superimposed for each $x$-position. The mean deviation of these profiles is about 1% for both the mean velocity and for the intensity suggesting that the mean flow is essentially two dimensional. This is later confirmed by a very small residue of
the divergence $\partial U/\partial x + \partial W/\partial z$ obtained from the PIV results (about 0.001 s$^{-1}$). Also, the LDV measurements reveal that the free-stream turbulence intensity, defined by $1/(\sqrt{u'^2/U} + \sqrt{w'^2/W})$, is less than 1%, such that freestream flow can be regarded as potential.

### 2.2 Roughness elements

The roughness elements are constituted by LEGO blocks 31.7 mm long, 15.7 mm wide and 9.6 mm high (Fig. 1b) to create a fully rough regime ($Re_z = u_z h/\nu \approx 200$). The arrangement (Fig. 1c) was chosen to represent a medium roughness length, $z_0$, the roughness parameter usually used in the atmospheric context and defined by the logarithmic law:

$$\frac{\bar{U}}{u_*} = \frac{1}{\kappa} \ln\left(\frac{z - z_d}{z_0}\right)$$

(1)

where $\kappa$ is the Von Kármán’s constant, fixed at 0.4, and $z_d$ the zero-plane displacement length. The roughness length is a hydrodynamic quantity related to the drag on the roughness elements, which is usually estimated via purely geometrical parameters in terms of the plane area density ($\lambda_p = A_p/A_t$) and the frontal area density ($\lambda_f = A_f/A_t$) (e.g. Raupach et al. 1980; Coceal and Belcher 2004). $A_t$ is the area of the reference domain (here $D_x \times D_y$), $A_p$ is the plane area of the obstacles viewed from above and contained in the domain of reference and $A_f$ is the frontal area of each obstacle exposed to the wind (see Fig. 1b, Fig. 1c and Table 1 for the dimensions).

For later comparison with the measured data and for the initial design of the surface layout, several models to estimate the roughness length $z_0$ as well as the displacement length $z_d$, based on the above mentioned geometrical densities, were evaluated. The results, based on the models of Kondo and Yamazawa (1986), Bottema (1995; 1997), Lettau (1969) and Macdonald et al. (1998), are given in Table 2. The estimations of $z_d$, proposed by Bottema (1995; 1997) and Macdonald et al. (1998), where an empirical drag coefficient is also taken into account, are similar. The purely geometrical methods, developed by Kondo et Yamazawa (1986) and Lettau (1969) seem to over-estimate the modelling of $z_0$ in comparison with those of Bottema (1995; 1997) and Macdonald et al. (1998). The latter values of $z_0$, are close to the desired value of $z_0/\delta$ for a medium roughness surface.

### 2.3 PIV measurements

The PIV experiments were performed at a free-stream velocity incident on the roughness element, $\bar{U}_{\infty,0}$, of 0.35 m s$^{-1}$. In order to characterize the upstream flow and the boundary-layer development over the rough surface,
measurements were performed at six x-positions. The flow fields, 0.5 m high and 0.6 m wide in order to capture and exceed the maximum boundary-layer depth, were centered at \( x = [0.2, 1, 3, 6, 8, 11] \) m with \( Re_x = \frac{U_\infty x}{\nu} = [5 \times 10^4, 3 \times 10^5, 1 \times 10^6, 2 \times 10^6, 3 \times 10^6, 4 \times 10^6] \), respectively. The images were taken in the center of the channel \((y = 0 \text{ m})\) with a X-STREAM VISION 10 bit CMOS camera with a 1260 x 1024 pixel resolution and equipped with a NIKKOR camera lens with a 50 mm focal length. Image acquisition was controlled by a frequency generator which allows the acquisition of \( N \) pairs of images at 1 Hz. A typical time interval between the two images of a pair, \( \Delta t \), was about 0.017 s. A high number of pairs was chosen to assure convergence of the turbulent statistics \((N = 999)\). The water was seeded with 60 \( \mu \text{m} \) polyamide particles (Orgasol \( \text{©} \ 2000 \)) illuminated with a laser light sheet 5 mm thick, created with an oscillating mirror and a 25 Watt Spectra Physics Beamlok Argon-Ion laser.

The PIV images were processed with the algorithms and interface developed by Fincham and Speckling (1997) and Fincham and Delerce (2000), which permitted to effectively eliminate the peak-locking bias errors. A resolution of 2 pixels mm\(^{-1}\) with a correlation box size of 20 pixels and a grid spacing of 10 pixels in the x-direction, and a correlation box size of 15 pixels and a grid spacing of 8 pixels in z-direction, yields a longitudinal resolution of about 5 mm and a vertical resolution of 4 mm, respectively. This resolution is roughly equivalent to the laser light thickness, thus giving an essentially isotropic measurement resolution (without significant loss of out-of-plane moving particles, estimated at less than 10 \%, well below the limit of 30 \% suggested by Fincham and Speckling (1997)).

Longitudinal spatial averaging of the measured velocity fields at a given x-position was performed to further increase statistical convergence. It is, however, only justified for the further downstream x-positions \((x > 3 \text{ m})\). At the beginning of the rough surface \((x = 0.2 \text{ m})\), the velocity field is subdivided into three parts (smooth, transition, rough) and x-averaged for each one. The fluctuations \((u'_i)\) in each case are the difference between the raw field \((u_i)\) and the spatially and temporally averaged field \((\overline{U}_i)\) which is given by:

\[
\overline{U}_i(z) \equiv \frac{1}{N} \sum_{p=1}^{N} \left[ \frac{1}{n_x} \sum_{q=1}^{n_x} u_i(q,z,p) \right],
\]

where \( N \) is the number of computed velocity fields \((N = 999)\) and \( n_x \) is the number of velocity vector columns used \((n_x = 90, \text{ for a complete field})\).

The integral time scale can be estimated to be larger than the advection time (i.e., Taylor’s hypothesis with \( u'/U << 1 \) where advection dominates). Therefore, a conservative estimate based on the maximum advection time and the sampling frequency of 1 Hz yields a roughly 50 \% overlap of the large scales between consecutive instantaneous velocity fields, or about 2000
independent samples at large scales. This is larger than the 1000 large scales suggested by Tennekes and Lumley (1972) for example to ensure statistical convergence. This sampling is higher than the Direct Numerical Simulations (DNS) of Coceal et al. (2006) who averaged over only 400 large scales. It should also be noted that the increased spatial dispersion as the top of the roughness elements is approached (i.e. the roughness sublayer), prevents representative mean statistics to be evaluated with single vertical plane measurements. If meaningful statistics in the roughness sublayer were desired, several vertical planes across a typical roughness area would need to be measured to allow spatial (double-) averaging in all horizontal planes (x, y). This was not the objective of this study, so the measurements and statistical evaluations were limited to above the roughness layer (taken as z > 2z_h, e.g. Macdonald 2000).

3 Boundary-layer development

3.1 Mean velocity field

In this section, the mean velocity fields as the boundary layer develops over the rough surface are presented. The log-law scaling parameters used to normalize the velocity and length quantities are also determined.

Fig. 2a-b show the temporally averaged longitudinal and vertical velocity fields, \( \overline{U}(x, z) \) and \( \overline{W}(x, z) \), respectively, around the beginning of the rough bed (x = 0 m). In the longitudinal velocity field \( \overline{U}(x, z) \) for x > 0, the development of the rough boundary layer can be seen. It can also be seen that the incident smooth boundary layer is relatively thin, as desired, of the order of the roughness elements \( (\delta_{smooth}/z_h \approx 1) \). The ensuing flow can thus be considered as a new boundary layer which starts to develop on the roughness elements under a potential and uniform free stream, as opposed to under an existing boundary layer. The effect of the surface roughness discontinuity, at x = 0, can clearly be seen in the strong \( \overline{W} \) perturbation centred around x = 0, with a strong positive increase in \( \overline{W} \) from effectively 0 to about 0.03 m s\(^{-1}\). This rise is symmetric with respect to the discontinuity at x = 0 m, with a radius of influence of about 0.18 m or about 10z_h. Also, since around the discontinuity the flow over the obstacles must first accelerate as it is deviated, it causes an initial acceleration of \( \overline{U} \) (\( \partial \overline{U}/\partial x > 0 \)) with an accompanying boundary-layer height decrease, as is indeed observed in Fig. 2a. Further downstream (x > 0.03 m), the new boundary-layer growth dominates due to the drag exerted, and \( \partial \overline{U}/\partial x < 0 \) while \( \partial \delta/\partial x > 0 \).

Fig. 3 shows the mean longitudinal velocity profiles obtained via time and spatial averaging at each measurement station (A-F). Because of the finite height of the water in the flume, the boundary-layer growth is associated
with a slight increase of the free-stream velocity ($U_\infty$) to balance the flow rate due to the increasing displacement thickness in the boundary layer. This increase in $U_\infty$ is weak, about 5% over all measurement stations, and in agreement with the preliminary LDV measurements (Sect. 2.1). The mean streamwise velocity profile in the log-law region given by Eq. 1, can be used to determine the roughness length $z_0$ and the friction velocity $u_*$. To evaluate the displacement height $z_d$ independently, we chose Macdonald’s (2000) semi-empirical proposition:

$$z_d = \left[ 1 + \alpha^{-\lambda_p}(\lambda_p - 1) \right] z_h$$

(3)

where $\alpha$ is an empirical coefficient, $C_d$ the drag coefficient fixed at 1.2 with a corrective factor $\beta$. For cubical and staggered obstacles, Macdonald et al. (1998) give $\alpha = 4.43$ and $\beta = 1$ which yields $z_d = 0.76 z_h$. Macdonald (2000) has verified the results of Raupach et al. (1980) and concluded that the log-law region, characterized by Eq. 1, is valid from $z = 2.3 z_h$ to $z = 3.5 z_h$. Thus, by linear regression in the region $2.3 z_h < z < 3.5 z_h$, for $x = [6, 8, 11]$ m, $u_*$ and $z_0$ was obtained (Table 3). The roughness length $z_0$ and the friction velocity $u_*$ for all values of $x > 6$ m were found to be approximately constant: 0.25 mm $\pm$ 0.05 mm and 0.02 m s$^{-1}$ $\pm$ 0.001 m s$^{-1}$, respectively. The value of $z_0$ agrees with the semi-empirical prediction established by Macdonald et al. (1998) reported earlier in Table 2. For all values of $x < 6$ m (positions A to C), estimating $u_*$ and $z_0$ with the log-law is not expected to work since the log-law region is too small for relatively large $z_h/\delta$, as discussed earlier.

Another way to determine $u_*$, independent of the log-law, is by assuming $u_*^2 = -\bar{w}'w''_{z=z_d}$. To evaluate the Reynolds stress in the canopy at $z = z_d$ which was not measured at this height, we can either extrapolate the linear stress variation in the outer layer or the essentially constant-stress part below ($z/\delta < 0.2$), as shown in a typical Reynolds stress profile in Fig. 4a. Here, we chose to take an average of the obtained minimum and maximum $u_*$ values, shown in Fig. 4b (squares) with those obtained from the log law (triangles). It can be seen that the friction velocity decreases with fetch until an equilibrium value is reached, as already observed by Rao et al. (1974) for a boundary layer over a change of surface roughness. In the case of a newly developing boundary layer under a freestream flow, a detailed discussion of the evolution of $u_*$ is given by Castro (2007).

It can be noted that the flow is fully rough as desired, since $z^+ = \frac{z u_*}{\nu} \geq 2$, or equivalently $k_s^+ = \frac{k_s u_*}{\nu} \geq 70$ with $k_s = z_0/0.033$ (e.g. Jiménez 2004).
3.2 Vertical development of the boundary layer

The boundary-layer height, \( \delta(x) \), has been determined via two methods. One is the height where \( \overline{U} \) reaches 99% of the free-stream velocity \( (\overline{U_\infty}) \). The other is the height where the flux \( -\overline{uw} \) decreases to 5% of its maximum value. The boundary-layer heights based on these two definitions and normalized by \( z_0 \) are shown in Fig. 5. They can be seen to be in close agreement although the height based on the mean velocity is consistently higher than the height based on the turbulent flux, a sign of both the robustness and the arbitrary nature of both criteria. This lead to the use of the average of both criteria to fit the evolution of the boundary-layer height, yielding:

\[
\frac{\delta}{z_0} = 0.18 \left( \frac{x}{z_0} \right)^{0.79} - 203. (4)
\]

As expected, this law differs from the classic smooth turbulent boundary-layer growth rate which predicts a power law exponent of 0.85 and a coefficient about 0.08 (White 1991). Our power law exponent is close to the well-known 4/5th power law, however, starting with Elliot’s model (1958), to predict the growth of an internal boundary layer developing after a sudden change of roughness. Numerous experimental studies of developing internal boundary layers (e.g. Antonia and Luxton 1971; Pendergrass and Arya 1984) as well as numerical ones (e.g. Lin et al. 1997) have yielded exponents close to 0.8. Townsend (1966) theoretical analysis shows that for a large change in friction velocity, very smooth to very rough, the flow behaves essentially as that of a boundary layer developing below a uniform free-stream velocity, i.e. the situation studied here. However, the coefficient and depth of the boundary layer measured is less than predicted by Elliot’s theory (1958) which is valid for small changes in friction velocity. Indeed, our measured coefficient of 0.18 is less than the 0.68 predicted by Elliot with an estimated smooth roughness length of \( z_{01} \) of \( 3 \times 10^{-5} \) m (Perrier 1988). Also, our coefficient is still smaller than the 0.35 predicted by Pendergrass and Arya (1984) for a roughness change parameter \( M = \ln \left( \frac{z_{01}}{z_{02}} \right) = -2.3 \) which is close to our \( M = -2.1 \).

3.3 Turbulent quantities

We now turn our attention to the evolution of the turbulent quantities. Fig. 6a-c show the profiles of \( \overline{u'^2}/u_s^2 \), \( \overline{w'^2}/u_s^2 \) and \( -\overline{uw}'/u_s^2 \), respectively, as a function of \( z/\delta \) where \( \delta \) is the local mean value of the boundary-layer height (Fig. 5) and \( u_s \) is the local value obtained from the Reynolds stress profile, for all measured \( x \)-positions. The normalized intensity of these variances and covariances, at last initially, increases with \( x \). According to the study of Antonia and Luxton (1971) again of an internal boundary-layer growth,
the increase of $\frac{u'^2}{u^*2}$ is linked to the intensification of the vertical gradient of $\overline{U}$ which is accompanied with an increase of dynamical production. Here, the stresses’ growth with $x$ tends to decrease until an equilibrium is reached where the profiles overlap. This equilibrium is reached between $x = 3$ m and $x = 6$ m ($Re_x = 1 \times 10^6$ and $2 \times 10^6$) for all three stresses.

Both $\frac{u'^2}{u^*2}$ and $-\frac{u'w'}{u^*2}$ are maximum near the top of roughness elements. This feature can be related with the existence of turbulent structures which are generated by the roughness elements (Kim et al. 1987; Castro et al. 2006). The existence of these structures is supported by the experimental work of Hommena and Adrian (2003). From these maxima, $\frac{u'^2}{u^*2}$ decreases linearly up to the top of the outer layer. The decrease of $-\frac{u'w'}{u^*2}$ is quasi-linear, as expected for a pressure driven flow. Yet, near the rough wall the shear stress remains roughly constant as usually observed for neutral rough boundary-layer measurements (Cheng and Castro 2002a and 2002b; Chow et al. 2005; Drobinski et al. 2007). This might be attributed, as the roughness elements are approached, to a lack of sufficient spatial resolution in this high intensity region and also some possible remaining spatial dispersion not accounted for in single plane measurements as the roughness layer is approached.

Fig. 6b shows that the $\frac{w'^2}{u^*2}$ profiles in equilibrium (for $x > 6$ m) have a maximum near $z/\delta \approx 0.3$ but vary very weakly between $0 < z/\delta < 0.5$ around a value of about 1. The quasi-constancy of $\frac{w'^2}{u^*2}$ until $0.5\delta$ agrees with the profiles observed by Panofsky (1974), Yaglom (1991) and Castro et al. (2006) and simulated by Drobinski et al. (2007) and Cocca et al. (2006). Near the surface, ($z/\delta \approx 0.1$), the normalized variances lie between 4 and 4.5 for $\frac{u'^2}{u^*2}$ and around 0.8 for $\frac{w'^2}{u^*2}$. These values are slightly lower than those deduced from the in-situ measurements and the LES results of Drobinski et al. (2007) which lie between 5 and 6 for $\frac{u'^2}{u^*2}$ and 1 to 2 for $\frac{w'^2}{u^*2}$. Fig. 6a and 6b reveal that $\frac{u'^2}{u^*2}$ is about four times smaller than $\frac{w'^2}{u^*2}$, i.e. the flow is strongly anisotropic, at large scales. It is also the ratio established by Moeng and Sullivan (1994) or the observations of Nicholls and Readings (1979) and Grant (1986) who measured $\frac{w'^2}{u'^2} \approx 4$.

It can be concluded that this experiment captures the turbulent structure of a developing rough neutral boundary layer, which reaches equilibrium between $Re_x \approx 1 \times 10^6$ and $2 \times 10^6$ ($3 < x < 6$ m). In order to investigate the transfer processes governing such a developing boundary layer, the budgets of the turbulent kinetic energy $e$, $\overline{u'^2}$, $\overline{w'^2}$ and $\overline{u'w'}$ are determined in the next section.
4 Normalized budgets of the second-order turbulent quantities

This section examines the budgets of the second-order turbulent transport equations to help evaluate the numerical models and characterize the transfers between the turbulent quantities. The Reynolds-stress transport equation is given by Eq. 5 where Coriolis and thermal production effects are not considered,

\[
\frac{\partial}{\partial t} (u'_i u'_j) = - \frac{U_k}{\partial x_k} \left( u'_k u'_i \frac{\partial U_j}{\partial x_k} + u'_k u'_j \frac{\partial U_i}{\partial x_k} \right) - \frac{1}{\rho_0} \left( u'_i \frac{\partial p'}{\partial x_j} + u'_j \frac{\partial p'}{\partial x_i} \right) - 2 \nu \left( \frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k} \right) - \frac{\partial u'_k u'_i u'_j}{\partial x_k} + \nu \frac{\partial^2 u'_i u'_j}{\partial x_k \partial x_k}. \tag{5}\]

The index \( k \) ranges from 1 to 3 and refers, respectively, to the longitudinal, transverse and vertical component. The terms on the right hand side represent: advection (\( \text{ADV} \)), dynamical production (\( \text{DP} \)), pressure-correlation (\( \text{PC} \)), dissipation (\( \text{DISS} \)), turbulent transport (\( \text{TR} \)) and molecular diffusion (\( \text{DIFF} \)). It may be noted that what is often referred to as the dissipation in Eq. 5 is in fact the pseudo-dissipation, \( \tilde{\varepsilon} \), defined by:

\[
\tilde{\varepsilon} = \nu \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_j}. \tag{6}\]

It can be related to the true dissipation, \( \varepsilon \) by:

\[
\varepsilon = \tilde{\varepsilon} + \nu \frac{\partial^2 u'_i u'_j}{\partial x_k \partial x_k}. \tag{7}\]

The diffusion term can be neglected since it is the product of the viscosity \((10^{-6} \text{ m}^2 \text{ s}^{-2})\) by second-order derivatives. As discussed in Sect. 2.1, under the present two-dimensional conditions, the transverse gradients \((\partial / \partial y)\) of the time-averaged quantities and the transverse mean velocity \((\overline{V})\) can also be neglected. Also, for \( x \)-stations at 1 m and further, the vertical mean velocity \((\overline{W})\) is negligible as discussed in Sect. 3.1. Finally, since the flow is statistically stationary, \( \partial u'_i u'_j / \partial t = 0 \).

4.1 Turbulent kinetic energy evolution equation

In addition to the above general approximations, in the budget of the turbulent kinetic energy \((e = u'_i u'_i / 2)\), the pressure-correlation term (\( \text{PC} \)) can
be neglected as it is expected to be a relatively weak transport term as for smooth boundary layers (Pope 2000). This implies:

$$PC_{u''}^2 + PC_{v''}^2 + PC_{w''}^2 \approx 0.$$  \hfill (8)

Eq. 5 applied to e, thus reduces to

$$0 = -U \frac{\partial e}{\partial x} - \frac{w''}{U} \frac{\partial U}{\partial x} - \frac{u''}{U} \frac{\partial U}{\partial z} - \varepsilon - \frac{\partial w'e}{\partial x} - \frac{\partial v'e}{\partial z}. \hfill (9)$$

\(\varepsilon\) cannot be estimated directly from the measurements, at least with reasonably accuracy, first, because the transverse gradients have not been measured and second, because the spatial resolution, 5 mm is too large. It is estimated to be about 16 dissipation scales (\(\eta\)) via the turbulence intensities in Fig. 6 and the later inteగre-scale computatiℌons (Sect. 5.1). This is not sufficiently small to include the peak of the dissipation spectrum, peaking at about 24\(\eta\) (Pope 2000). However, since \(\varepsilon\) is the only unknown quantity of the measured budget Eq. 9, it can be estimated via the residual.

Also, for evaluating the turbulent kinetic energy, \(\varepsilon\) an assumption needs to be made for \(v''^2\). The measurements of Cheng and Castro (2002b) and Macdonald et al. (2002) as well as the simulations of Castro et al. (2006) show that \(v''^2 \approx 0.5(w''^2 + w''^2)\), while the in-situ experiments of Drobinski et al. (2004, 2007) and the simulations of Moeng and Sullivan (1994) suggest that the coefficient is closer to 0.4. Here, the coefficient of 0.5 is retained since because the studies which support this approximation have a configuration close to our experiment, notably for the characteristics of the canopy (\(z_0/\delta_h\) and \(u_*/U_{\infty}\)). It can also be noted that this uncertainty on \(v''^2\) affects the estimation of the turbulent transport and the advection terms by a factor of 7\%, and the pseudo-dissipation term by about 4\%.

The turbulent kinetic energy budget is shown in Fig. 7a. Above \(z/\delta \geq 0.1\), advection is a minor term, even smaller than the also minor vertical turbulent transport term. However, the vertical turbulent transport is not negligible at the top of the boundary layer, for 0.6 < \(z/\delta\) < 1. Indeed, above \(z/\delta > 0.6\), although weak compared to the dissipation and the dynamical production in the lower layer, it becomes the only significant source here. Advection also grows in importance, becoming a minor sink term peaking near \(z/\delta \approx 0.8\). This balance between the turbulent transport and the advection agrees with the simulations of Mason and Thomson (1987) of the neutral boundary layer. In the outer layer, but below \(z/\delta \approx 0.6\), the dynamical production essentially balances the dissipation as observed by Glendening and Lin (2002) in LES of an internal boundary layer. The preponderant contribution of the dynamical production has also been assessed by Drobinski et al. (2004) and more recently by Castro et al. (2006) and Burattini et al. (2008). It is the classic assumption which stipulates that the production rate equals the dissipation.
It also supports the outer layer similarity hypothesis, the budgets being very similar as for smooth boundary layers (Pope 2000). Fig. 7b shows the longitudinal evolution of the dynamical production term of $e$, the most significant term of the budget of $e$ measured directly. It can be seen that for $x = 1$ m and $x = 3$ m (positions B and C) the dynamical production is higher than for the further $x$-positions in the range $0.3 < z/\delta < 0.7$. This source "surplus" is balanced by an increase in the sink of the vertical turbulent transport of $e$ (not shown) which decays by $x = 6$ m. By $x = 6$ m at the latest, all the profiles have converged (including the other terms, not shown) in accordance with the observation with regard to the turbulent stresses (Fig. 6a-c) that equilibrium is reached in the region $3$ m $< x < 6$ m.

### 4.2 $\overline{u'^2}$ evolution equation

For stationary and two-dimensional flows, Eq. 5 applied to $\overline{u'^2}$ reduces to

\begin{equation}
0 = -\frac{U}{\delta} \frac{\partial \overline{u'^2}}{\partial x} - 2 \left( \frac{\partial u'}{\partial x} \frac{\partial U}{\partial x} + \frac{u'w'}{\partial z} \frac{\partial U}{\partial z} \right) - \frac{2}{\rho_0} \frac{\partial \rho'}{\partial x} - \frac{\partial \overline{u'^2}}{\partial z} - 2 \nu \left( \frac{\partial u'}{\partial x}^2 + \frac{\partial u'}{\partial z}^2 \right) - \frac{\partial \overline{u'^2}}{\partial z} - \frac{\partial \overline{w'u'^2}}{\partial z} \right). \tag{10}
\end{equation}

The dissipation term ($\varepsilon_{u'^2}$) and the pressure-correlation ($PC$) term are the two unknown terms in the budget. Thus, in order to balance the budget, an additional hypothesis needs to be made. Pope (2000), based on DNS data of Spalart (1988) of a boundary layer over a flat smooth plate, shows that close to the wall, the anisotropy in the dissipation rate ($\varepsilon_{u'u_j}$) of the Reynolds stresses, $\overline{u'_i u'_j}$, is clearly large. However, for $z/\delta > 0.1$, there is approximate isotropy. The small level of anisotropy in $\varepsilon_{u'u_j}$ for $z/\delta > 0.1$ can be attributed to the relatively low Reynolds number of the DNS of Spalart (1988). For rough-wall boundary layers, the experiments of Saddoughi and Veeravalli (1994) and Saddoughi (1997) confirm that at this altitude there is also a local isotropy of $\varepsilon_{u'u_j}$ at high Reynolds numbers. Some studies even conclude that the roughness increases the degree of isotropy closer to the wall and this impact is greater with three-dimensional roughness (Castro et al. 2006). Therefore, as the present experiment was performed at high Reynolds number ($Re_u = \delta u_*/\nu > 3 \times 10^3$ at $x = 6$ m) and with three-dimensional roughness, the dissipation rate ($\varepsilon_{u'u_j}$) of $\overline{u'_i u'_j}$ can reasonably be expressed as

\begin{equation}
\varepsilon_{u'u_j} = \frac{2}{3} \varepsilon \delta_{ij}, \tag{11}
\end{equation}

where $\delta_{ij}$ is the Kronecker function. Thus, taking $\varepsilon_{u'^2} = 2/3\varepsilon$, with $\varepsilon$ obtained from Eq. 9, the pressure-correlation is the only residual term of the
budget of $u'^2$. The estimation of $\varepsilon_{u'^2}$ and consequently of the pressure-correlation term is thus again essentially affected by the uncertainty on $u'^2$ by a factor inferior to 4%.

The budget of $u'^2$ (Fig. 7c) shows, as expected, that the intensity of all the terms decreases as $z/\delta$ increases. The turbulence acts principally in the lower part of the outer layer. Above the surface layer, the dynamical production is the only significant source as shown by Pope (2000) for a smooth boundary layer. The advection by mean wind or turbulent transport are not really important for the evolution of $u'^2$, except at the top of the outer layer where it peaks near $z/\delta \cong 0.8$ and where the turbulent transport becomes the dominant source, as for the budget of $e$. The pressure-correlation term is significant throughout. Indeed, it is the main sink for $u'^2$ above the surface layer. Similarly to boundary layers over a smooth surface (Spalart 1988), the pressure fluctuations appear to distribute the energy between the different components: from $u'^2$ to $v'^2$ and $w'^2$. This is confirmed in the next section. Fig. 7d shows that equilibrium of the most significant term in the $u'^2$ budget, the dynamical production term, is reached between $x = 3$ m and $x = 6$ m, as for the budget of $e$ and in agreement with the longitudinal evolution of $\overline{w'^2}/u'^2$ (Sect. 3.3).

### 4.3 $w'^2$ evolution equation

For stationary and two dimensional flows, Eq. 5 applied to $w'^2$ reduces to

$$0 = - \frac{U}{\text{ADV}} \frac{\partial w'^2}{\partial x} - 2 \frac{\overline{w'p'}}{p_C} - 2 \nu \left( \left( \frac{\partial w'}{\partial x} \right)^2 + \left( \frac{\partial w'}{\partial z} \right)^2 \right) - \frac{\partial w'^2}{\partial x} - \frac{\partial w'^3}{\partial z}.$$  \hfill (12)

Here, the dynamical production term depends on $W$ and can thus be neglected. Also, as $\varepsilon_{u'^2}$ for the $u'^2$ budget, $\varepsilon_{w'^2}$ is estimated by Eq. 11. The pressure-correlation term ($PC$) is then again the only residual term of the measured budget of $w'^2$.

The $w'^2$ budget (Fig. 7e) shows that the turbulent transport term ($TR$) is again weak, peaking near $z/\delta \cong 0.8$ as a source similar to $u'^2$ and $e$ but smaller in magnitude. The advection, also weak, peaks near $z/\delta \cong 0.8$ as a sink similar to $u'^2$ and $e$. The main balance, below $z/\delta \cong 0.8$, is between the pressure-correlation and the dissipation term. In the outer layer, around $z/\delta = 0.8$, the main source for $w'^2$ is thus the pressure-correlation term as seen in the recent computations of Ashrafin and Andersson (2006). This observation confirms the previous hypothesis: the pressure fluctuations re-
distribute the energy from $u w'$ to $u w'$. It can also be observed that the $u w'$ pressure-correlation term attains half the absolute value of the $u w'$ pressure-correlation term, suggesting that the energy is equally partitioned between $u w'$ and $u w'$. This is confirmed by considering that the sum of the pressure-correlation terms of $u w'$, $u w'$ and $u w'$ is zero (Eq. 8) since the pressure-correlation term of $e$ is assumed to be zero (see Sect. 4.1), which allowed $PC_{v w'}$ to be evaluated. Yet, it is important to insist on the fact that the pressure-correlation terms of $u w'$ and $w' v'$ are individually and independently estimated: each one is the residue of their respective budget for the five downstream $x$-positions. Finally, Fig. 7f shows the longitudinal evolution of $PC_{w v'}$ term. In agreement with all other terms it can be seen that equilibrium is reached for $x > 3$ m.

Fig. 8a shows all three pressure-correlation terms at $x = 6$ m while Fig. 8b shows the ratio $-PC_{w v'}/PC_{w v'}$. It can be seen in Fig. 8a that $PC_{v w'}$ and $PC_{w v'}$ are well superposed for all $z/\delta$. The mean value of the ratio $-PC_{w v'}/PC_{w v'}$ in Fig. 8b reveals a ratio close to 2 for $0 < z/\delta < 0.8$. For $z/\delta > 0.8$, the ratio surprisingly decreases close to 1, but the magnitude of the terms is probably too small to get a significant estimation of the ratio. Awaiting more precise measurements for $z/\delta > 0.8$, it can thus be concluded that the pressure fluctuations equitably distribute the energy contained in the longitudinal fluctuations to the transverse and vertical fluctuations:

$$PC_{v w'} \cong PC_{w v'} \cong -\frac{1}{2} PC_{w v'} .$$

(13)

This redistribution, observed for all $x$-positions, is essentially the same as has been observed over smooth boundary layers (Pope 2000), suggesting that the turbulent structure in the outer layer is not influenced by the roughness elements.

### 4.4 $u w'$ evolution equation

For $u w'$, under the stationarity and 2D assumptions, Eq. 5 reduces to

$$0 = -\bar{u} \frac{\partial u' w'}{\partial x} - \left( u'^2 \frac{\partial \bar{u}}{\partial x} + u' w' \frac{\partial \bar{u}}{\partial z} \right)_{ADV}$$

$$- \left( \frac{1}{\rho_0} \left( u'^2 \frac{\partial \bar{u}}{\partial z} + w'^2 \frac{\partial \bar{u}}{\partial x} \right) - \frac{\partial u'^2 \bar{w}'}{\partial x} - \frac{\partial u' \bar{w}'^2}{\partial z} \right)_{TR},$$

where the dissipation term $\varepsilon_{u w'}$ has been neglected (see Eq. 11). The pressure-correlation term is thus the residual term here. The budget of $u w'$ is presented in Fig. 7g.

Fig. 7g shows that the advection and turbulent transport terms are again
relatively weak but with reversed signs compared to $\overline{u'^2}$, $\overline{w'^2}$ and $\epsilon$, and are still peaking near $z/\delta \approx 0.8$. Below, the main balance is between the dynamical production (source) and the pressure-correlation (sink), similar to the budget of $\overline{u'^2}$ where dissipation contributes as a sink. This result agrees with the measurements of Wyngaard (1992) and the simulations of Ashraafian and Andersson (2006). Wyngaard (1992) shows that the turbulent transport is negligible and that the pressure-correlation acts as a sink term which locally balances the dynamical production term. This repartition of energy was also observed by Mulhearn (1978) in wind-tunnel modelling of a rough-to-smooth internal boundary-layer type transition.

In conclusion, all the budgets reach equilibrium between stations at $x = 3$ m and $x = 6$ m while the distribution of the budget terms is very similar to smooth turbulent boundary layers and internal boundary layers.

5 Length scales

In this section we investigate the dissipative lengthscale $l_\epsilon$ and the mixing length $l_m$ based on the measured statistics, as well as the integral length-scales based on spatial correlations. An accurate quantification of these scales is essential as they drive the modelling of the flow. $l_\epsilon \equiv u^3/\epsilon$ and $l_m \equiv (K/\partial \overline{U}/\partial z)^{1/2}$ are functional length scales, based on the ratio of statistical properties of the flow (Hunt et al. 1989), the former often being used to estimate the latter which involves the sought-after Reynolds stress. Here, $u$ is a characteristic turbulent velocity, and $K$ is the turbulent eddy viscosity or exchange coefficient given by $K = -\overline{u'w'/(\partial \overline{U}/\partial z)}$ for horizontally homogeneous flow. First-order models are based on the so-called Prandtl-Kolomogorov relation, which assumes $K \equiv ul$, where $u$ is most often taken as $e^{1/2}$ and $l$ is a lengthscale of the energy containing eddies, well described by the integral scale, but usually taken from a functional lengthscale. The aim of this section is to characterize the turbulent scales of a neutral boundary-layer flow and to improve parameterization of the functional lengths.

5.1 Integral lengthscales

Integral lengthscales can be obtained directly from spatial correlations with the present PIV measurements, without relying on the often necessary Taylor hypothesis. In particular, the spatial correlation function in the $(x, z)$ plane,

$$R_{u'_i u'_j}(\vec{r}, \vec{\delta r}) = \frac{\overline{u'_i(\vec{r})u'_j(\vec{r} + \vec{\delta r})}}{u'_i(\vec{r})u'_j(\vec{r})},$$

(15)
can be estimated directly, where $\vec{r} = x\vec{t} + z\vec{k}$ is the reference position and $\vec{dr}$ taken as either $\delta x\vec{t}$ or $\delta z\vec{k}$ is the displacement, which is either positive or negative. The correlation functions are computed in each of the $N = 999$ fluctuating $(x, z)$ velocity fields with $\vec{r}$ for all measured points in the velocity fields. The resulting correlation functions are then averaged over all $N$ for convergence.

The integral lengthscales at each reference $x$-position are then computed via the integral of the correlation functions (Eq. 15):

$$L_{u_iu_j,\delta z}(\vec{r}) = \int_0^\infty R_{u_iu_j}(\vec{r}, \vec{dr})d\vec{dr}.$$  \hspace{1cm} (16)

To assure systematic convergence, the integration for large $r$ was stopped when the correlation function reaches 0.1.

The measurements permit 16 different integral scales to be obtained, i.e. for the four stresses ($u'^2$, $w'^2$, $u'w'$ and $w'u'$), each in two directions ($\delta x\vec{t}$ and $\delta z\vec{k}$), with $\delta x$ and $\delta z$ either positive or negative. As expected, it was observed that $\delta x$ either positive or negative does not impact on $L_{u_iu_j,\delta z}(\vec{r})$, underlining that the flow is homogenous in $x$, at least at the scale of the measurement fields. However, for $L_{u_iu_j,\delta z}(\vec{r})$, the results revealed a general tendency of larger integral scales on the top half of the boundary layer for negative $\delta z$ displacements than for positive ones, and vice-versa on the bottom half.

Since Carlotti and Drobinski (2004) argued that the smallest vertical integral scales should be close to the mixing length, the minimum integral-scale value between the positive and negative $\delta z$ displacements at each $z$-level (for all four stresses) was chosen. The resulting integral scales are regrouped in Fig. 9a-h for all $x$-positions. It can be seen that equilibrium of the integral scales is also reached between $3 \text{ m} < x < 6 \text{ m}$, as for the turbulent statistics and budgets. Nevertheless, the first $x$-positions ($x = 0.2 \text{ m}$ and $x = 1 \text{ m}$) exhibit the same tendencies except for $L_{u_iu_j,\delta x}$. It can also be noted that $L_{u_iu_j,\delta x}$ and $L_{u_iu_j,\delta z}$ reveal a greater gap between $x = 3 \text{ m}$ and $x = 6 \text{ m}$. This illustrates that the $u'$ fluctuations are more affected by the growing of the boundary layer than $w'$ which are more associated with the local turbulence. Also, it can be observed that $L_{u_iu_j,\delta z}$ dominates all other lengthscales, as expected. This dominance of $L_{u_iu_j,\delta z}$ is due to the existence of coherent structures which are elongated in the longitudinal direction (e.g. Drobinski et al. 2007). These structures are localized in the bottom of the boundary layer with a maxima at $z/\delta = 0.2$ which can also be observed in the $L_{u_iu_j,\delta z}(z)$ profiles. For 1D modelling purpose, it is more appropriate to consider the vertical scales, given by $\delta z$ displacements. As seen in the left column of Fig. 9, all the vertical integral scales increase linearly until a maximum value is reached around $L_{u_iu_j,\delta z}/\delta = 0.4$ in the range of $0.3 < z/\delta < 0.6$. Above, the vertical integral scales decrease to $L_{u_iu_j,\delta z}/\delta = 0.2$, $L_{u_iu_j,\delta z}/\delta = 0.2$ and $L_{u_iu_j,\delta z}/\delta = 0.1$ at $z/\delta = 1$. Only, $L_{u_iu_j,\delta z}$ increases again above $z/\delta = 0.9,$
which underlines the fact that significant exchanges of longitudinal structures occur between the boundary layer and the free stream. The vertical velocity fluctuation $L_{u'w'}\frac{\delta z}{\delta x}$ and $L_{w'u'}\frac{\delta x}{\delta z}$ are very close however, suggesting that the lengthscales of $w'^2$ are essentially isotropic.

Fig. 10a compares directly the profiles of the resulting vertical integral-scales of the four stresses. Here, it can be seen more clearly that the vertical integral lengthscales essentially have the same shape and intensity, increasing in the bottom half of the boundary layer and decreasing above. More precisely, below $z/\delta \leq 0.2$, in the inertial sublayer, all profiles collapse very well, but above there are some differences. Also, as expected due to the inhomogeneity in the $z$-direction, the integral scales of the $u'w'$ and $w'u'$ fluxes are not equal, with a marked shift in the maximum scale.

To compare the integral lengthscales with $l_m$ and $l_\varepsilon$ used in 1D prediction models at equilibrium, a representative integral lengthscale ($L_z$) was taken as the average of the four vertical lengthscales in Fig. 10a. This mean integral lengthscale ($L_z$), shown in Fig. 10b, is linear in the lower part of the outer layer, as expected. Above, $L_z$ tends to be hyperbolic. The maximum value reached by integral scale is $L_z/\delta \cong 0.3$ at $z/\delta = 0.5$. A function of the form $az/(b + z^k)$ is thus expected to be adapted, as the dashed line in Fig. 10b shows. The fitted normalized equation is:

$$\frac{L_z}{\delta} = \frac{A \left( \frac{z - z_h}{\delta} - s \right)}{B + \left( \frac{z - z_h}{\delta} - s \right)^p}, \quad \text{(17)}$$

with $A = 0.26; s = 0.05; p = 2.4$ and $B = 0.24$.

### 5.2 Dissipative lengths

In the following sections, we consider only the region where the flow has reached equilibrium, i.e. for $x > 6$ m. Taylor (1935) first suggested that $\dot{\varepsilon} \cong u^3/l$, usually written as:

$$\dot{\varepsilon} = C_\varepsilon \frac{e^{3/2}}{l_\varepsilon}, \quad \text{(18)}$$

where $C_\varepsilon$ is a constant largely fixed at 0.845 (Schmidt and Schuman 1989; Cuijpers and Duynkerke 1993; Canuto et al. 2001; Cheng et al. 2002). Using the dissipation rate obtained from the budget of $e$ (Eq. 9), $l_\varepsilon$ can thus be determined via Eq. 18. Fig. 11a shows the resulting vertical profile of $l_\varepsilon$ normalized by the experimental integral length $L_z$ of Fig. 10b. Clearly, except between $0.3 < z/\delta < 0.7$, where $l_\varepsilon \cong 2.4L_z$, $l_\varepsilon/L_z$ is not constant. This is not surprising since the two scales should only be proportional in homogeneous turbulence.
5.3 Mixing length

The mixing length model,
\[ \overline{u'w'} = -C_m l_m^2 \left| \frac{\partial \overline{U}}{\partial z} \right| \frac{\partial \overline{U}}{\partial z}, \tag{19} \]
where \( C_m \) is a constant, is used to model the Reynolds stress when one lengthscale dominates. It can be used here to deduce the mixing length \( l_m \) from the measured mean velocity gradient and Reynolds stress profile, also for \( x > 6 \) m.

Fig. 11b shows the resulting normalized profile \( l_m C_m^{1/2} / L_z \). In comparison with the dissipative length, \( l_\epsilon / L_z \), the mixing length \( l_m \) matches \( L_z \) better. Neglecting the strong nonlinearities near the surface for \( z \leq 0.1 \delta \), the scales are approximately proportional over a wider range, between \( 0.3 < z/\delta < 0.9 \), with a ratio of proportionality \( (l_m C_m^{1/2} / L_z) \) of about 0.37.

The standard mixing length in the logarithmic layer,
\[ l_m = \kappa (z - z_d), \tag{20} \]

in combination with \( l_m C_m^{1/2} \) determined from Eq. 19, permits \( C_m \) to be determined in the log-law region via linear regression, yielding an average \( C_m = 0.51 \) for the three latest \( x \)-stations. Thus, the ratio of proportionality \( l_m / L_z \) in the zone \( 0.3 \leq z/\delta \leq 0.9 \) is about 0.52. Fig. 12 shows \( l_m / \delta \) with \( C_m = 0.51 \) as well as the standard mixing length in the logarithmic layer (Eq. 20). The formulation of Blackadar (1962),
\[ l_m = l_0 \frac{\kappa z}{\kappa z + l_0}, \tag{21} \]
has been also given in Fig. 12, by determining \( l_0 \) by a least squares fit of \( l_m \) for the last three \( x \)-stations. The resulting three \( l_0 / \delta \) ratios are very close, with an average of 0.24. It can be seen that the normalized measurements for all three stations follow essentially the same curve, which might be described as a weak S. In the central outer-layer, between \( 0.3 \leq z/\delta \leq 0.7 \), \( l_m / \delta \) is largely constant at about 0.13, before retreating slightly and then increasing as the top of the boundary layer is approached. Not surprisingly, Blackadar’s relation used for all stability conditions does not fit the data well. A parametrization based on either the dissipative scale or the vertical integral scale would also yield significant differences, so direct parametrization of the mixing length based on the measured \( l_m \) profiles in Fig. 12 might be most appropriate.
6 Conclusion

The main goal of this study was to investigate the turbulence of the neutral boundary layer to help validate numerical simulation models and to help improve the parameterizations of the turbulent processes for one-dimensional neutral boundary-layer models. The experiment presented in this paper reproduces the development of a neutral boundary layer in a water flume over a rough surface at high Reynolds numbers. The velocity measurements were performed via a two-dimensional particle image velocimetry (PIV) technique in the vertical symmetry plane of the flow. The turbulent statistics at several longitudinal positions were obtained by spatially and temporally averaging the PIV velocities fields, assuring statistical convergence for the higher-order statistics.

The boundary-layer growth was established on the basis of the mean longitudinal velocities and the turbulent shear stress, yielding a 0.8 power law in agreement with previous results for the growth of internal boundary layer. However, it appears that our 0.18 coefficient is smaller than the one of an internal boundary layer. The beginning of the roughness surface is well revealed by mean vertical velocity perturbation whose streamwise extend is about 0.18 m.

An analysis of the budget of turbulent kinetic energy ($e$) shows that shear production is the main source of turbulence in the outer layer, as for turbulent boundary layers over smooth surface. The residual term of this budget yielded the dissipation rate. This allowed also the dissipation of the $u'^2$ and $w'^2$ budgets to be estimated. Finally, we were able to quantify the pressure fluctuations and describe its role in transferring energy between the different variances. Again, these transfers were found to mirror the exchanges due to the pressure-correlation terms in smooth boundary layers, suggesting the turbulent structure of the outer layer over a rough surface is very similar to one over a smooth surface. The development of the rough neutral boundary layer does not significantly modify the repartition between the constitutive terms of the budget of $e$, $u'^2$, $w'^2$ and $u'w'$. Whatever the abscissa, even before equilibrium is reached, the same terms govern the transfer of energy, except for the vertical turbulent transport of $e$ which is a minor term for $0.1 < z/\delta < 0.6$ at all stations except for $x = 1$ m where it is not negligible, and where it appears as a sink of energy which balances the "surplus" of dynamical production. All budgets as well the first and second order statistics reach equilibrium between $12000 \leq x/z_0 \leq 24000$.

Finally, the spatially resolved measurements allowed us to compute the spatial integral lengthscales directly, including the vertical ones, more appropriate for modelling purposes. These integral length were used to compare with the dissipative and the mixing lengths functional scales. It is shown that the ratio of the dissipative scale to vertical integral lengthscale, not expected to be constant in this inhomogeneous flow, indeed varies significantly by a fac-
tor up to 2. Direct calculation of the mixing length suggest a better match with the vertical integral scale, with a narrow constant of proportionality for \( z/\delta \geq 0.1 \) of about \( l_m/L_z \cong 0.52 \). Further numerical tests need to be done in order to validate the new proposal of \( L_z \) and verify whether it improve the modelling of neutral boundary layer.

**Acknowledgements**

This work study was supported by the MesoScale Meteorology and Experiment Meteorology groups (GMME and GMEI) of the National Center of Meteorological Research of Météo-France. The authors would like to thank B. Beaudoin, J-C. Boulay, J-C. Canonici, M. Morera, S. Lassus Pigat and H. Schaffner of the CNRM-GAME fluid mechanics laboratory (SPEA) for their support during the experiments for providing us with valuable comments and availability.
Table 1: Dimensions of the reference domain (streamwise $D_x$ and crosswise $D_y$), the spacing between the roughness elements contained in the domain (streamwise $W_x$ and crosswise $W_y$), the dimensions of the roughness elements (height $z_h$, length $L_t$ and width $L_s$), the different areas (the area of the reference domain $A_t$, the plane area of the obstacles viewed from above and contained in the domain of reference $A_p$ and the frontal area of each obstacle exposed to the wind $A_f$) and densities (the plane area density $\lambda_p$ and the frontal area density $\lambda_f$) (all units in mm).

<table>
<thead>
<tr>
<th>DOMAIN</th>
<th>SPACINGS</th>
<th>OBSTACLES</th>
<th>AREAS</th>
<th>DENSITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_x$</td>
<td>$D_y$</td>
<td>$W_x$</td>
<td>$W_y$</td>
<td>$z_h$</td>
</tr>
<tr>
<td>32</td>
<td>61.3</td>
<td>8</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>$L_t$</td>
<td>$L_s$</td>
<td>$A_t$</td>
<td>$A_p$</td>
<td>$A_f$</td>
</tr>
<tr>
<td>32</td>
<td>16</td>
<td>1962</td>
<td>1024</td>
<td>480</td>
</tr>
<tr>
<td>$\lambda_p$</td>
<td>$\lambda_f$</td>
<td>0.5</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Roughness parameters $z_d$ and $z_0$ and ratio $z_d/z_h$ and $z_0/z_h$ obtained from the predictions based on analytical theories.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_0$ (mm)</td>
<td>1.1</td>
<td>1.2</td>
<td>0.01</td>
<td>0.25</td>
</tr>
<tr>
<td>$z_0 / z_h$</td>
<td>0.11</td>
<td>0.12</td>
<td>0.001</td>
<td>0.025</td>
</tr>
<tr>
<td>$z_d$ (mm)</td>
<td>NA</td>
<td>NA</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>$z_d / z_h$</td>
<td>NA</td>
<td>NA</td>
<td>0.66</td>
<td>0.76</td>
</tr>
</tbody>
</table>
Table 3: Estimation of the roughness parameters, $z_0$ and $u_*$ by linear regression of the log-law in the region $2.3z_h < z < 3.5z_h$ and for $x > 6$ m with $z_d = 7.6$ mm, for $U_{\infty,0}=0.35$ m s$^{-1}$.

<table>
<thead>
<tr>
<th>$x$ (m)</th>
<th>6</th>
<th>8</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_*$ (m s$^{-1}$)</td>
<td>0.020</td>
<td>0.021</td>
<td>0.019</td>
</tr>
<tr>
<td>$z_0$ (mm)</td>
<td>0.253</td>
<td>0.346</td>
<td>0.223</td>
</tr>
</tbody>
</table>
Figures

Fig. 1: (a) Sketch of the experimental set-up. (b) Typical element, LEGO©. (c) Arrangement of the roughness elements.

Fig. 2: Mean velocity fields at position A \((x = 0 \text{ m})\), near the beginning of the rough bed (a) \(\mathbf{U}(x, z)\), (b) \(\mathbf{W}(x, z)\).

Fig. 3: Mean longitudinal velocity profiles \(\mathbf{U}(z)\) at all measurement \(x\)-positions (A-F).

Fig. 4: (a) Method used to estimate the friction velocity \(u^*_s\) from the \(\mathbf{u}'\mathbf{w}'\) profiles: the minimum value is obtained by averaging the first points of the profile \((z/\delta < 0.2)\) and the maximum value is given by the intersection between the extrapolation of the linear regression of \(\mathbf{u}'\mathbf{w}'\) for \(0.5 < z/\delta < 0.9\) and \(z = z_d\). (b) Squares symbols: friction velocity \(u^*_s\) estimated by method (a) with \(u^*_s = (u^*_s + u^*_s)/2\) via \(-\mathbf{u}'\mathbf{w}'\) shear stress profiles and Triangle symbols: friction velocity \(u^*_s\) estimated via the log-law fits in the region \(2.3z_h < z < 3.5z_h\) (see Table 3).

Fig. 5: Boundary-layer height evolution \((\delta)\) normalized by the roughness length \((z_0)\), the square symbols: 5\% of \(\mathbf{u}'\mathbf{w}'_{\text{max}}\) criterion; the X symbols: 99\% of \(\mathbf{U}_\infty\) criterion; the triangle symbols: average of \(\delta_{\mathbf{u}'\mathbf{w}'_{5\%}}\) and \(\delta_{\mathbf{U}_\infty}\); the solid line: the best fit of the average of both criteria.

Fig. 6: Locally normalized profiles of (a) \(\mathbf{u}'\mathbf{u}'/u^*_s^2\), (b) \(\mathbf{w}'\mathbf{w}'/u^*_s^2\) and (c) \(\mathbf{w}'\mathbf{u}'/u^*_s^2\) as a function of \(z/\delta\) for all measured \(x\)-positions.

Fig. 7: Locally normalized budget of \(e\) (a), \(\mathbf{u}'\mathbf{u}'\) (c), \(\mathbf{w}'\mathbf{w}'\) (e) and \(\mathbf{u}'\mathbf{w}'\) (g) as a function of \(z/\delta\) for the D position \((x = 6 \text{ m})\). Dominant source terms of each budget: the dynamical production term \((DP)\) for the budget of \(e\) (b), \(\mathbf{w}'\mathbf{w}'\) (d), \(\mathbf{u}'\mathbf{w}'\) (h) and the pressure-correlation term \((PC)\) for the budget of \(w'^2\) as a function of \(z/\delta\) for all \(x\)-positions.

Fig. 8: (a) Locally normalized magnitude of the pressure-correlation terms \(PC\) of the budget of \(\mathbf{u}'\mathbf{u}'\), \(\mathbf{w}'\mathbf{w}'\) and \(\mathbf{u}'\mathbf{w}'\) as a function of \(z/\delta\) for the D position \((x = 6 \text{ m})\). (b) Ratio of the normalized pressure-correlation terms \(-PC_{\mathbf{u}'\mathbf{u}'}/PC_{\mathbf{w}'\mathbf{w}'}\) as a function of \(z/\delta\) for the D position \((x = 6 \text{ m})\).

Fig. 9: Normalized minimum vertical and longitudinal integral length-scale \(Lr_{u'_j,\delta z}(z)\), in the left column and \(Lr_{u'_j,\delta x}(z)\), in the right column, for all \(x\)-positions for \(u'^2\) (a and b), \(\mathbf{u}'\mathbf{u}'\) (c and d), \(\mathbf{w}'\mathbf{w}'\) (e and f) and \(\mathbf{w}'\mathbf{u}'\) (g and h).
Fig. 10: (a) Normalized minimum vertical integral length scales for \( \overline{u'^2} \), \( \overline{w'^2} \), \( \overline{u'w'} \), and \( \overline{w'u'} \). (b) Mean vertical integral length scale \( (L_z) \) of (a) and its fit with \( \frac{L_z}{\delta} = \frac{A(z - \frac{z_h}{\delta} - s)}{B + (\frac{z - z_h}{\delta} - s)^p} \), with \( A = 0.26; \ s = 0.05; \ p = 2.4 \) and \( B = 0.24 \).

Fig. 11: (a) Ratio of the integral length scale and the dissipative length \( (l_\varepsilon / L_z) \) at developed \( x \)-positions as a function of \( z/\delta \). (b) Ratio of the integral length scale and the mixing length \( (l_mC_{m}^{1/2} / L_z) \) at developed \( x \)-positions as a function of \( z/\delta \).

Fig. 12: Measured mixing length \( l_m/\delta \) deduced from the Eq. 19 (symbols) at developed \( x \)-positions as a function of \( z/\delta \), compared to the Eq. 20, dashed line, \( l_m = \kappa (z - d) \) and Eq. 21, solid line \( l_m = l_0 \frac{\kappa z}{\kappa + \nu \kappa} \) (Blakadar, 1962).
Figure 1: (a) Sketch of the experimental set-up. (b) Typical element, LEGO®. (c) Arrangement of the roughness elements.
Figure 2: Mean velocity fields at position A (x = 0 m), near the beginning of the rough bed (a) $\bar{U}(x, z)$, (b) $\bar{W}(x, z)$.
Figure 3: Mean longitudinal velocity profiles $U(z)$ at all measurement $x$-positions (A-F).

Figure 4: (a) Method used to estimate the friction velocity $u^* = \frac{-\overline{u'w'}}{\overline{u'^2}}$ from the $u'w'$ profiles: the minimum value is obtained by averaging the first points of the profile (for $z/\delta < 0.2$) and the maximum value is given by the intersection between the extrapolation of the linear regression of $u'w'$ for $0.5 < z/\delta < 0.9$ and $z = z_d$. (b) Squares symbols: friction velocity $u_*$ estimated by method (a) with $u_* = \frac{(u_{*\text{max}} + u_{*\text{max}})}{2}$ via $-\overline{u'w'}$ shear stress profiles and Triangle symbols: friction velocity $u_*$ estimated via the log-law fits in the region $2.3z_h < z < 3.5z_h$ (see Table 3).
Figure 5: Boundary-layer height evolution ($\delta$) normalized by the roughness length ($z_0$), the square symbols: 5 \% of $u'w'_{max}$ criterion; the X symbols: 99 \% of $U_\infty$ criterion; the triangle symbols: average of $\delta_{u'w'_{5\%}}$ and $\delta_{U_{99\%}}$; the solid line: the best fit of the average of both criteria.
Figure 6: Locally normalized profiles of (a) $\frac{u'^2}{u_*^2}$, (b) $\frac{w'^2}{u_*^2}$ and (c) $\frac{-u'w'}{u_*^2}$ as a function of $z/\delta$ for all measured $x$-positions.
Figure 7: Locally normalized budget of $e$ (a), $u'^2$ (c), $w'^2$ (e) and $u'w'$ (g) as a function of $z/\delta$ for the D position ($x = 6$ m). Dominant source terms of each budget: the dynamical production term ($DP$) for the budget of $e$ (b), $u'^2$ (d), $u'w'^2$ (h) (which equals the $DP$ for the budget of $e$) and the pressure-correlation term ($PC$) for the budget of $w'^2$ as a function of $z/\delta$ for all $x$-positions.
for $0 < z / \delta < 0.8$: the average of the ratio is 2.

Figure 8: (a) Locally normalized magnitude of the pressure-correlation terms $PC$ of the budget of $u'^2$, $w'^2$ and $v'^2$ as a function of $z/\delta$ for the D position ($x = 6$ m). (b) Ratio of the normalized the pressure-correlation terms $-PC_{u'^2}/PC_{w'^2}$ as a function of $z/\delta$ for the D position ($x = 6$ m).
Figure 9: Normalized minimum vertical and longitudinal integral lengthscale $L_{\overline{w'w}, \delta z}(z)$, in the left column and $L_{\overline{w'u}, \delta x}(z)$, in the right column, for all x-positions for $\overline{u'^2}$ (a and b), $\overline{w'^2}$ (c and d), $\overline{u'w'}$ (e and f) and $\overline{w'u'}$ (g and h).
Figure 10: (a) Normalized minimum vertical integral lengthscales for $u'^2$, $w'^2$, $u'w'$ and $w'u'$. (b) Mean vertical integral lengthscale ($L_z$) of (a) and its fit with $L_z = \frac{A(z - z_h) - s}{B + (\frac{z - z_h}{\delta} - s)p}$, with $A = 0.26; s = 0.05; p = 2.4$ and $B = 0.24$.

Figure 11: (a) Ratio of the integral lengthscale and the dissipative length ($l_\varepsilon/L_z$) at developed $x$-positions as a function of $z/\delta$. (b) Ratio of the integral lengthscale and the mixing length ($l_m C_m^{1/2}/L_z$) at developed $x$-positions as a function of $z/\delta$. 

Tomas S., Eff O. and Masson V. (2011) Experimental investigation of the turbulent momentum transfers in a neutral boundary layer over a rough surface, Boundary-Layer Meteorology, 138, 385-411

author-produced version of the final draft post-refeering
the original publication is available at www.springerlink.com - doi : 10.1007/s10546-010-9566-0
Figure 12: Measured mixing length $l_m/\delta$ deduced from the Eq. 19 (symbols) at developed $x$-positions as a function of $z/\delta$, compared to the Eq. 20, dashed line, $l_m = \kappa(z - d)$ and Eq. 21, solid line $l_m = l_0 \frac{\kappa z}{\kappa z + l_0}$ (Blakadar, 1962).
REFERENCES


Blackadar AK (1965) A single layer theory of the vertical distribution of wind in a baroclinic neutral atmospheric boundary layer. Final report, aercr-65-531, Department of Meteorology, The Pennsylvania State University, University Park, PA, 22 pp


Bouttier F (2003) The arome mesoscale project. In: Seminar on Recent Developments in Data Assimilation for Atmosphere and Ocean, 8-12 September, ECMWF, European Center for Medium-Range Weather Forecasts

Bouttier F (2007) Arome, avenir de la prévision régionale. La Météorologie 58:12–20


41


