Identifying statistical properties of solar radiation models by using information criteria

Laurent Linguet, Yannis Pousset, Christian Olivier

Identifying statistical properties of solar radiation models by using information criteria

Laurent Linguet[a*], Yannis Pousset[b], and Christian Olivier[b]

[a] Laboratoire UMR Espace-DEV, Université de Guyane
275 route de Montabo, BP 165, 97323 Cayenne Cedex, Guyane Française
laurent.linguet@univ-guyane.fr

[b]Equipe RESYST (REseaux et SYStèmes de Télécommunication), Université de Poitiers
Institut XLIM UMR CNRS n°7252, 86962 Futuroscope Cedex, France
yannis.pousset, christian.olivier@univ-poitiers.fr

ABSTRACT

The purpose of this article is to improve modeling of statistical properties of solar radiation models through the analysis of measurement data on the ground in the intertropical zone. For this, we identify, using information criteria, the probabilistic distributions introduced in two models of synthetic solar radiation generation. We then validate the results by using the KL divergence and KSI parameter as comparison criteria between the distributions arising from real and synthesized data. Our study confirms, for example, that the Gaussian classical distribution is not suitable for modeling solar radiation, and we propose other more suitable statistical laws instead. The value of the identification procedure of the distribution laws presented in this article is that it ensures the generation of solar radiation data comparable in their statistical content to the measured data. Another advantage is that this procedure contributes to highlighting the time invariance of distribution laws representing the random terms. We conclude that this new information-criteria-based method permits the identification of the probability laws that best describe the statistical distributions introduced in the models of synthetic solar radiation generation

KEYWORDS: information criteria, synthetic solar irradiance model, model selection.

---

1. Introduction

Knowledge of solar radiation, or irradiance, on the surface of the Earth is of great interest in many fields. Climate sciences require reliable and sufficiently numerous solar data for understanding climate change. Agriculture and natural ecosystems, in general, are affected by solar irradiance, and it is therefore necessary to study it to understand the current impact of

---

In this paper, we use the abbreviations *IC* for information criteria, *BIC* for Bayesian information criterion, *MDL* for minimum description length, *TAG* for time-dependent autoregressive Gaussian, *pdf* for probability density function, *cdf* for cumulative density function, *KL* for Kullback–Leibler, and *KSI* for Kolmogorov–Smirnov Integral.

climate change (Stanhill and Cohen, 2001). In terms of energy, the design and sizing of systems using solar energy input (such as solar water heaters, photovoltaic cells, or solar thermal concentrators) require solar data to simulate and test their long-term energy efficiency (Mellit et al., 2008). In architecture, simulating the energy performance of buildings in urban areas also requires solar irradiance data (input data) to size additional clean energy production systems (solar thermal, photovoltaic, etc.) that are able to meet heating and electricity needs while optimizing the total energy consumption of buildings (Amado and Poggi, 2012). In all these areas, as in others, solar irradiance data represented by medium and long-term time series are often necessary.

The time series of solar irradiance can be obtained from data measured by either ground measuring stations or satellites (Marie-Joseph et al., 2013), or by selecting periods of representative measurement data and calculating an irradiance average year, called a typical meteorological year (Bilbao et al., 2004).

However, these methods, although simple, have disadvantages. In the first case, the time series are limited to reproducing historical data and do not reproduce the full variability range of irradiance data; they are sometimes also liable to be incomplete. In the second case, there is no guarantee that the developed typical meteorological year includes the statistical characteristics of the long-term climate of the chosen location, or that it reproduces the extreme values of irradiance. Finally, another disadvantage is that in many areas the low density of ground measuring stations and/or the absence of measures derived from satellite irradiance data precludes the use of these methods. Most of the time, all these constraints require resorting to temporal series of irradiance generated synthetically at different temporal resolutions (hours, days, etc.) depending on the user's needs.

During the past two decades, several methods have been developed to synthetically generate time series of solar irradiance. All models must meet the following condition (Hansen et al., 2010): produce series with the same statistical content as that of time series observed over a particular locality. In addition, the simulated irradiance values must be statistically consistent with those that have been measured. Some of the best known methods include:(1) using an autoregressive moving average originally developed by Graham and Hollands (1988) and improved by Collares and Pereira (1992),and also used by other authors (Muselli, 1998;Tiba and Fraidenraich, 2004); (2) using a Markov model associated with a transition matrix (Markov transition matrix model) developed by Aguiar et al. (1988) and taken forward by other authors, including, notably, the use of neural networks to configure the transition matrix of the Markov model (Linares-Rodríguez, 2011; Poggi, 2000); (3) the method developed by Boland (1995), which combines the autoregressive model with Fourier analysis; and (4) a method developed by Polo (2011) to model a time series from a mean value to which a random fluctuation is added, whose characteristics (amplitude and frequency) depend on cloud conditions of the sky.

The first three methods use, for convenience, a Gaussian distribution to represent the term of the modeling error (random term). This was the choice for most of the tools that help determine the parameters of autoregressive (or autoregressive moving average) models, because in the field of time series analysis, assuming the random term as a Gaussian white noise is a simplifying condition for determining the model. However, in reality, solar irradiance time series are physically confined; that is, they contain only positive values and cannot exceed a maximum value corresponding to the extraterrestrial solar irradiance (solar irradiance measured above the atmosphere). Therefore, the Gaussian distribution does not perhaps provide the best representation of the error associated with the models, as shown by Boland (2008).

The last method, Polo's model (Larrañeta, 2015, Polo, 2011),, uses a beta distribution to model the random fluctuation of the irradiance (the random term). However, in the literature, in addition to the beta distribution, there are also other distributions (Boltzmann, gamma, and exponential) that approximate the probabilistic laws of solar irradiance time series.

The purpose of this paper is therefore to determine the probabilistic laws best describing the statistical distribution of each random term impacting two well-known synthetic solar irradiance generation models: Aguiar and Pereira's (1992) and Polo's (2011).

To determine these distributions, we will introduce a law selection method based on *IC* (Alata et al., 2013), which is also called the generalized entropic criteria and will be preferred to the classical maximum likelihood. From several criteria, we choose the *BIC* (Schwarz, 1978) and the Φ_β (El Matouat and Hallin, 1996) criteria on account of their strong consistency (almost certain convergence). They generally require a fairly large amount of data, which we do have, as will be shown later, in the experimental phase.

This remainder of the paper is organized as follows.

- In Section 2, we recall the statistical and physical concepts used later: the *IC* and the generation of solar irradiance time series.
- In Section 3, we describe the law identification process between various proposal laws corresponding to the random term laws introduced in the generation models, and also describe the validation process.
- In Section4, we present the results obtained from the law identification process in the previous section and also select the laws best representing the random terms.
- In section 5, we compare synthetic irradiance data generated from the selected laws with measured data, in order to analyze the performance of synthetic models and validate or invalidate the conclusions of the law identification process.

2. Concepts from statistics and physics

2.1. The information criteria (*IC)*

The *IC* (for a state of the art, see, for example, Olivier and Alata (2009)) are tools that provide a partial response to the parsimony problem: given a set of realizations or data or observations $x^N = (x_1, ..., x_N)$ of a random process *X*, and given Θ, a family of parametric models chosen *a priori*, what is the model $\hat{\theta}$ of Θ that best corresponds to the process *X*? In other words, we are looking for the number and values of the free parameters of a model $\hat{\theta}$, the optimal model in the sense of the *IC*.

The concept of IC pertains to assigning each competing model θ_l of Θ a penalty term "offsetting" the classical log-likelihood term $L(\theta_i)$, which involves minimizing the expression

$$IC(i) = L(\theta i) + |\theta i|.C(N) \tag{1}$$

where $C(N)$ is a term usually dependent on N, the number of observations; and $|\theta_i|$ is the number of free parameters of the model θ_i. Recall that the only criterion of maximum likelihood (satisfied here by minimizing $L(\theta_i)$) is insufficient when $|\theta_i|$ varies because the maximum likelihood leads to an over-parameterization.

The expression of the penalty $|\theta_i|.C(N)$ is obtained by minimizing the cost between models, in general of the f-divergence as KL or Bayesian stochastic complexity, and differs according to criteria. The best-known and oldest one is the Akaike criterion (Akaike, 1974) but it is inconsistent. In our study, we select only the Schwarz (1978) and El Matouat and Hallin (1996)criteria, denoted by BIC and Φ_β, respectively, and we exclude the other criteria that are not strongly consistent, that is, not almost surely convergent when $N \to +\infty$ (see Olivier and Alata (2009)). For example, Hannan and Quinn's (1979) criteria, denoted by Φ, is weakly consistent (convergence in probability).

Regarding BIC, we have: $C(N) = \log N$, and for Φ_β, we have $C(N) = N^{\beta} \log\log N$, with $0 < \beta < 1$.

Let us note that the MDL criterion(Rissanen,1989), well-known in information theory (arithmetical binary encoding of compression norms), is also strongly consistent, although it differs from BIC only by negligible terms when N is large; which is why we consider only the BIC criterion. The Φ_β criterion can be seen as a compromise between the BIC/MDL and Φ criteria.

Regarding Φ_β, Jouzel et al. (1998) have shown that we have a finer condition,

$$\beta_{\min} = \frac{\log\log N}{\log N} < \beta < 1 - \beta_{\min},$$

to warrant strong consistency, and that it has been shown (see Olivier and Alata., 2009) that $\beta_{\min}$ is the best value of β in numerous applications.

Let us note that one of the advantages of Φ_β is that it generalize the whole criteria by one expression; thus, the limit case $\beta = 0$ corresponds to Φ, whereas the β solution of $n^{\beta} \log\log N = \log N$ corresponds to the BIC/MDL criteria (Alata et al., 2013).

Finally, the inequality $IC(\theta_i) < IC(\theta_j)$ means the model θ_I achieves a better compromise between adequacy to data, as measured by likelihood $L(\theta_i)$,and the cost of that choice of model, as measured by $|\theta_i|.C(N)$. Therefore, θ_I is selected over model θ_j. The successful model $\hat{\theta}$ is therefore the one that minimizes the IC criteria:

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}_i \in \Theta}{\arg\min}[IC(\boldsymbol{\theta}_i)] \tag{2}$$

In this paper, these criteria will be applied to the selection of models of probability laws.

2.2. Generation of synthetic time series of solar irradiance

Surface solar irradiance (global irradiance) can be represented as a combination of two components: deterministic and stochastic. The former represents the daily and seasonal irradiance variations and can be described by the well-established astronomical equations that describe the position of the sun relative to the latitude and longitude of the location being studied. The stochastic component is the result of random events that affect surface solar irradiance, such as the frequency and height of clouds, their optical properties, and the turbidity of the atmosphere linked to its composition (aerosol, water vapor, ozone contents, etc.)

The standard procedure for modeling a time series of synthetic irradiance from a time series of irradiance measurements is to eliminate the contributions of the deterministic components, to make the series stationary, and then attempt to model the time series of the stochastic term. Once the stochastic term has been modeled, simply reintroduce the deterministic component to obtain a synthetic time series.

To isolate the stochastic component, we either use the methods based on the Fourier analysis (Linares-Rodríguez et al., 2011) (one then proceeds by subtracting the frequency contributions of the deterministic component), or the clearness index k_t (Aguiar and Collares-Pereira, 1992; Bilbao, 2004; Graham, 1988; Hansen, 2010, Polo, 2011).The clearness index k_t is the ratio of the overall ground irradiance G and the extraterrestrial I_{oh} global irradiance on a horizontal plane:

$$k_t = G / I_{oh} \tag{3}$$

$$I_{oh} = I_{sc} E_0 \cos\theta_z \tag{4}$$

where I_{sc} is the irradiance produced by the solar constant, E_0 is the correction factor of eccentricity, and θ_z is the solar zenithal angle.

Eccentricity defines the shape of the Earth's elliptical orbit around the sun; and characterizes the flattening of the Earth's ellipse with respect to a circle.

The solar zenith angle at any given location is the angle between the straight line from the ground location to the sun and the perpendicular direction to the surface of the place considered (zenith).

The clearness index compares the irradiance measurements taken at different times without losing information on the amplitude of the irradiance and is then denoted by $k_t(h)$, where h is the time considered. The clearness index, k_t, can be defined for different time intervals on hourly, daily, and monthly bases.

Graham et al. (1988) find that seasonal variations in daily radiation are due to changes in the extraterrestrial radiation, and these seasonal variations can be captured by the use of clearness index. This finding enabled the development of many algorithms that synthesize radiation data at a finer scale from measurements at a larger scale (monthly or yearly). Although clear sky index (in the limit of a perfect clear sky model) generates a truly stationary time series we do not use it because the present study investigates the random term impacting two synthetic solar irradiance generation models that synthesize solar radiation data at a finer scale from measurements at a larger scale. As part of our study, we consider two types of models using k_t.

2.2.1. The *TAG* model

Aguiar and Collares-Pereira's (1992) *TAG* model generates synthetic hourly irradiance data using an autoregressive model, not homogeneous in time, and assumes a Gaussian distribution. As its sole input, it uses the monthly average of the daily clearness index, denoted by K_T. The wide availability of the monthly average data worldwide makes it an easily usable model. The *TAG* model has the advantage of being flexible enough to model the main features of solar irradiance, and accurate enough to be used in energy applications. The study of the sequential properties of solar irradiance by Aguiar and Collares-Pereira (1992) has shown that it essentially depends on the value of the irradiance from the previous hour, which led them to propose the following model:

$$y(h) = \phi(K_T).y(h\text{-}1) + r_{TAG}(h) \qquad (5)$$

$$\phi(K_T) = 0.38 + 0.06\cos(7.4K_T - 2.5) \qquad (6)$$

This is an autoregressive (Equation 5) model where $r_{TAG}(h)$ is the random term whose distribution we are seeking to identify, h is the hourly variable time, $\phi(K_T)$ is a correlation coefficient depending on the K_T index (see equation (6), and $y(h)$ (see equation (6)) is the normalized clearness index. Normalization of the clearness index $k_t(h)$ provides a highly stationary time series. The resulting model fits the data from different measurement sites: it is the invariance of the probabilistic law relative to the localization.

Normalizing $k_t(h)$ is carried out according to the following expression:

$$y(h) = \frac{k_t(h) - k_{tm}(K_T, h)}{\sigma(K_T, h)} \qquad (7)$$

where $k_{tm}(K_T,h)$ is the hourly average of $k_t(h)$, and $\sigma(K_T, h)$is the standard deviation of $k_t(h)$.Both depend on the monthly average clearness index K_T.

We calculate $k_{tm}(K_T,h)$ as follows (Aguiar and Collares-Pereira, 1992):

$$k_{tm}(K_T, h) = \lambda(K_T) + \varepsilon(K_T)(-\frac{\kappa(K_T)}{\sin(h_j)}) \qquad (8)$$

where

$$\lambda(K_T) = -0.19 + 1.12K_T + 0.24\exp(-8K_T) \tag{9}$$

$$\varepsilon(K_T) = 0.32 - 1.6(K_T - 0.5)^2 \tag{10}$$

$$\kappa(K_T) = 0.19 + 2.27K_T{}^2 - 2.51K_T{}^3 \tag{11}$$

$$\sigma(K_T, h) = A.\exp(B.(1 - \sin(h_j)) \tag{12}$$

$$A = 0.14.\exp(-20(K_T - 0.35)^2 \tag{13}$$

$$B = 3.(K_T - 0.45)^2 + 16.K_T{}^5 \tag{14}$$

and h_j is the solar hourly angle.

2.2.2. Polo's model

Polo et al. (2011) allow a time series to be modeled from a mean value and standard deviation of random values. This method generates a synthetic time series of the clearness index, k_t, from measurements of an average clearness index, k_{tm}, over a given period. One of the main conditions imposed by the method is that the frequency and amplitude of fluctuations in synthetically generated irradiance values are statistically representative of real conditions, that is, the function(s) of distribution of the original data is (are) comparable to that of synthetically generated data.

The method to generate synthetic hourly clearness index values involves adding two contributions: that of the average for the period (time) considered and of the stochastic fluctuations around this average. Mathematically, the expression of the synthetic clearness index at a time h can be formulated as follows:

$$k_t(h) = k_{tm}(j) + A(h).\text{sign}(s) \tag{15}$$

where $k_{tm}(j)$ is the average daily value of the clearness index for day j, *A(h)* is the random amplitude of the fluctuation of the clearness index for the hour h, *A(h)* is also the random term whose distribution we are seeking to identify, sign(s) is the sign of the random signal and s is the realization of a normal Gaussian distribution centered with zero mean and standard deviation unit.

The hourly average of irradiance, $k_{tm}(j)$, can be obtained from the data measured *in situ*, and the second term of equation (12), *A(h).*sign*(s)*, can be generated using the following procedure:

- Generate standard deviation values from a distribution f (e.g., by pulling random numbers from a uniform distribution and finding the corresponding value of the inverse distribution f^{-1}).
- To obtain *A(h)*,multiply the generated standard deviation values by the maximum value of the measured standard deviations.
- Generate a random signal s and assign the sign of s, sign(s), to *A(h)*, and add the result to the average value $k_{tm}(j)$.

For Polo's model, the analysis of the probability distribution of process *A(h)* is conducted under different types of skies. We create three classes of average clearness indices corresponding to three types of sky cloudiness: cloudy, partly cloudy, and clear sky (C_1 to C_3classes, respectively). The thresholds for the three classes were chosen in order to obtain a number of measurements that are approximately equivalent in each of the classes:

- C_1:(cloudy sky): $k_{tm} \leq 0.42$;
- C_2: (partly cloudy sky): $0.42 < k_{tm} < 0.54$;
- C_3: (clear sky): $k_{tm} \geq 0.54$.

2.3. Data used

To identify and validate the distribution laws of the random terms r_{TAG} and A, we use hourly irradiance data from the Meteo France agency measured by two ground weather stations located at Rochambeau and Ile Royale in French Guyana. The weather stations measure hourly means of global irradiance on a horizontal plane and have Kipp and Zonen pyranometers of type CM6B, equipped with a ventilation fan. The CM6B instruments fulfill the accuracy requirements of a secondary standard pyranometer defined in WMO (2008), which are specified as 3%. Meteo France provides preventive maintenance every two months (e.g., cleaning of the air filter and the glass dish, desiccant exchange). Standard exchange of the pyranometers is systematically carried out every two years. Each pyranometer is calibrated in the Radiometry National Center of Meteo France located in Carpentras, France. Once installed, the coefficients of the new pyranometer are then entered into the data acquisition system of the *in situ* station. .

The two stations are located on the Atlantic coast as shown in Figure 1. The Rochambeau station is located at an altitude of 4m above the sea level and the surrounding area includes the Félix Eboué airport. The Ile Royale station is located at an altitude of 48m on the island of the same name located in the Atlantic Ocean, 14 km away from the coast of French Guyana. *In situ* observations have an hourly temporal resolution and are issued at each hour (HH00 UTC-3). The clearness indices for these two sites have been obtained with the extraterrestrial hourly irradiance data calculated at the top of the atmosphere. The data time range spans the years 1996 to 2010, on a daily basis, from 7 a.m. to 5 p.m.

The radiation database was divided into two phases: for law identification and for testing the selected laws. For the first phase, we choose data from:

- the Royale station over four years (1998, 2002, 2007, and2009); and
- the Rochambeau station over seven years (1996, 1998, 2000, 2003, 2005, 2007, and 2009).

This gives us data of the order of $O(10^5)$.

For the phase of law validation, we choose measured data independent of that used for identification (base of learning) of the laws (see §.3.1). Therefore, we use data from:

- the Royale site over four years (1999, 2006, 2008, and 2010); and

- the Rochambeau site over seven years (1997, 1999, 2002, 2004, 2006, 2008, and 2010).

3. Methods

3.1. Identification process

The purpose of the identification process is to determine the probabilistic laws best describing the statistical distribution of each random term impacting the two synthetic solar irradiance generation models: Aguiar and Pereira's (1992) and Polo's (2011).We assume that both random terms are independent and identically distributed (i.i.d.) and that we can produce a time series. We extract, from the measured data of the identification phase (section 2.3), the time series of the random terms r_{TAG} and A as follows:

- $r_{TAG}(h)$ hourly values are obtained from equation (5), by computing $y(h)$and $\phi(K_T)$ with the monthly average of daily clearness index values, K_T, and the solar hourly angle values, h_j , obtained from the in situ solar irradiance data.
- $A(h).sign(s)$ values are obtained from equation (12).

The identification of the probability distribution best describing the random terms is made on the basis of candidate laws:

- For the *TAG* model, in view of the literature and the shape of the probability laws, we select as candidate laws only the Gauss, Logistic and Extreme Value laws.
- Unlike the previous model, nine laws with one or two parameters are tested for the Polo's model: Rayleigh and exponential, for the laws with one parameter; and Beta, Gamma, lognormal, inverse Gaussian, Rice, Nakagami, and Weibull, for the laws with two parameters.

We refer to the Appendix for the expression of these 12 laws.

Each candidate law defines a model θ_l of a model family Θ according to the *TAG* or Polo models. It is therefore a matter of two model selection problems in which the best process A or r_{TAG} is sought for generating the two variables $y(h)$ and $k(h)$with $|\theta_l|$ being the number of free parameters of the considered probabilistic distribution of the random terms r_{TAG} or A. In reality, the penalty term has an influence only on Polo's model (one or two parameters following the nine candidate laws), so for the *TAG* model, with the three candidate laws with two settings, only log-likelihood term $L(\theta_l)$ is influential.

To determine the best probability distributions describing the statistical distribution of r_{TAG} and A, we use a law selection method based on the *BIC* criteria (Schwarz, 1978) and the Φ_β criteria. The steps of the identification process are listed below:

- For each model and class, we divide the random terms (r_{TAG} and A) into 100 packets of 10000 values.

- We use the candidate laws to calculate both information criteria (*BIC* and Φ_β) for all the 100 packs and compute their average value.
- We then seek the minimum average values and determine the best candidate law for each model and class.

3.2. Validation process

The validation process seeks to assess the goodness of the best candidate laws selected with the *IC*. First, synthetic hourly means of solar irradiance have been simulated on the 11 years reserved for the phase validation (section 2.3) by using the best candidate laws to generate the random terms as follows:

- Identify the parameters of the candidate distributions by generating the time series of random terms (r_{TAG} and $A(h)$) as shown in the identification procedure described above and by using in situ data;
- Generate synthetic series of random terms (r_{TAG} and $A(h)$) with the parameters of the candidate distributions obtained previously;
- Calculate the synthetic clearness index:
  - for TAG model by using equations (5) and (7);
  - for the Polo's model by using equation(15)after calculating the average daily clearness index of each day by using the in situ data;
- Compute the synthetic solar irradiance data from equation (3) by multiplying the synthetic clearness index by extraterrestrial irradiance, which gives:

$$\mathrm{G_{synt}}\,(\mathrm{h}) = \mathrm{k_{t_{synt}}}\,(\mathrm{h}) * \mathrm{I_{oh}}\,(\mathrm{h}) \tag{16}$$

Second, we compare the *pdf*s and *cdf*s between the measured solar irradiance (i.e., observed) and the synthetic solar irradiance generated from the candidate laws. Comparisons are made by using *KSI* parameter between *cdf*s and *KL* divergence between *pdf*s.

3.2.1. KSI parameter

In order to compare the similarities between *cdf*s of synthetic solar irradiance and *cdf* of measured solar irradiance we used a variant of the *KS* test: the Kolmogorov-Smirnov test Integral (*KSI*) developed by Espinar et al. (2009). The *KS* test originally defines a *D* statistic as the maximum value of the absolute difference between two *cdf*s. The belonging Null hypothesis can be formulated as follows: if the *D* statistic is lower than a threshold value *Vc*, the two data sets could statistically be the same and in this case the Null hypothesis is accepted. However the application of the *KS* test only materializes in the acceptance or rejection of the Null hypothesis and it cannot be used to compare several *cdf*s each other with respect to a reference *cdf*. The *KSI* test define the *D* statistic over *n* intervals (n = 100) of the entire data range ([x_{min} ,.., x_{max}]). So instead of getting one value *D*, we get a series of *n* values D_n and the *KSI* parameter is defined as the integral of the series D_n. Then the *KSI* parameter is the integral of the differences between the *cdf*s of two sets of data. More detailed information on the method can be found in Espinar et al. (2009). The *KSI* parameter formulation is:

$$KSI = \int_{x_{min}}^{x_{max}} D_n \, dx \quad (17)$$

The minimum value of the *KSI* parameter is zero, which means that the *cdf*s of the two sets compared are equal. We used the *KSI* parameter to compare the similarities between *cdf*s of synthetic solar irradiance generated from the candidate laws and *cdf* of measured solar irradiance. We stated that the recognized *cdf* is the one that gets the lowest *KSI* parameter (the one that best fits the *cdf* of measured solar irradiance). We repeated the comparisons 500 times on N = 10.000 generated and measured samples. The recognition percentage of each candidate law is given by the following ratio :

$$recognition\ percentage = \frac{\text{number of recognitions}}{\text{number of experiences}}$$

3.2.2. Kullback-Leibler divergence

The Kullback-Leibler divergence, D_{KL}, measures the difference between two probability densities (*pdf*) f_x and f_y. f_x is the probability density of a measured data while f_y is the theoretical density of a statistical law. In practice, the calculation of D_{KL} is done with the histogram H constructed from the measured data and the theoretical probability density f_y. If we denote by *H(x)* the height of one class of the histogram H at a point x, the *KL* divergence from this class is calculated by an integral (H(x) - fy(x)).log(H(x)/fy(x)) on the interval [b_l; b_u], where b_l and b_u are the lower and upper limits of the class. Thus, for a k classes histogram, D_{KL} is given by the following equation:

$$D_{KL}(H, y) = \frac{1}{2} \sum_{i=1}^{k} \int_{b_l}^{b_u} \left(H(x) - f_y(x)\right) \log\left(\frac{H(x)}{f_y(x)}\right) \quad (18)$$

In order to assess the goodness of the best candidate laws selected with the *IC* we computed the *KL* divergence for these candidate laws, and we stated that the recognized *pdf* is the one that gets the lowest *KL* divergence (the one that best fits the in situ *pdf*). We repeated the comparisons 500 times on N = 10.000 generated and measured samples. The recognition percentage of each candidate law is given by the same ratio formula as for *KSI*.

4. Results

We present the identification results in Tables 1 and 2, for both models and their candidate laws. The values shown in Table 1 and Tables 2ato 2c are those with the average values of *IC*; 100 packets of N = 10000 data are considered, regarding random terms observed on an hourly basis variable h.

In case of the *TAG* model (Table 1), results are consistent concerning both *IC*. We find the traditionally admitted Gaussian distribution law in the top three, but in the second position. The most adequate law, in the sense of our selection criteria, is the logistic law, and the least adequate is the extreme value law.

Table 1: Average values of criteria for all three candidate laws, regarding the TAG model.

	Gaussian	*Logistic*	*Extreme value*

BIC	***35985***	***28045***	*44771*
$\Phi_{\beta_{\min}}$	***35973***	***28033***	*44759*

In the case of Polo's model (Tables 2a to 2c), after the *IC* test, we selected only the laws in the top five best recognition rates. The negative values of the *IC* criteria in Table 2 are justified by the impulsive nature (at maximum amplitude >> 1) of candidate laws. The *BIC* or $\Phi_{\beta_{\min}}$ criteria selected the same top five laws, regardless of the cloudiness class. These are the Nakagami, Weibull, Beta, Gamma, and exponential laws. We ignore the other four laws in all three tables. We note that for both models, the *BIC* and $\Phi_{\beta_{\min}}$ criteria provide the same ranking, which is normal because of the high quantity of considered data (N = 10000), remember that both criteria are consistent (almost certain convergence). Neither criteria offers better benefit than the other. In Tables 2a to 2c, the results are quite different for the different classes of cloudiness; and this is justified by the variability of the model according to the intensity of solar irradiance. Regarding the C_1 and C_2classes, the beta and Nakagami laws are clearly distinguishable from the Weibull law, whereas the latter differs a little from the gamma law (in the top two) but differs strongly from the other three candidates in the case of low cloudiness (class C_3).

Table 2: Average values of criteria for the laws in the top five, according to the three classes of cloudiness for Polo's model.

(a) class C_1

	Beta	Weibull	Exponential	Gamma	Nakagami
BIC	-14 897	-14 748	-14 130	-14 572	-14 938
$\Phi_{\beta_{\min}}$	-14 915	-14 767	-14 139	-14 570	-14 957

(b) class C_2

	Beta	Weibull	Exponential	Gamma	Nakagami
BIC	-9 965	-9 664	-8 527	-9 335	-9 944
$\Phi_{\beta_{\min}}$	-9 983	-9 682	-8 537	-9 354	-9 962

(c) class C_3

	Beta	Weibull	Exponential	Gamma	Nakagami
BIC	-11202	-11577	-10883	-11507	-11285
$\Phi_{\beta_{\min}}$	-11220	-11596	-10893	-11526	-11595

5. Validation of models

To discuss the previous findings, the validation method described above has been used to assess the goodness of the best candidate laws. In view of the *IC* values obtained at the stage when laws are selected, we apply the validation method on all the five candidate laws and present only the more significant results, those of the laws adopted in the top two. In Tables 3 and 4, we present, regarding the *TAG* model and then Polo's model, the recognition percentages of each of the top two laws following the considered *KL* distance and *KSI* parameter.

In the case of the *TAG* model, results in Table 3 confirm the findings of Table 1. These results show that the *IC*-based method permits to identify the probabilistic distributions introduced in the *TAG* model. We can therefore accept that a white logistical random term yields to a best representation of the error associated with the model proposed by Aguiar and Collares-Pereira (1992) rather than a Gaussian one, which would confirm Polo's findings (2011). This method also contributes to the highlighting of the time invariance of distribution laws representing the random term.

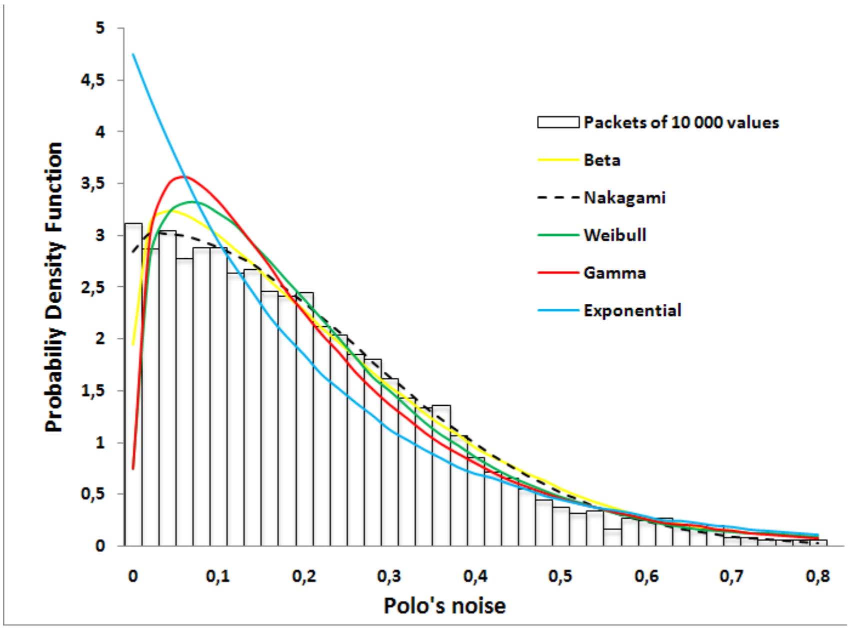
Table 3: Recognition percentage according to the KSI parameter and KL distances between measured and generated laws for the TAG model.

	Gaussian	*Logistic*
KSI	***35%***	***65%***
KL	***34%***	***66%***

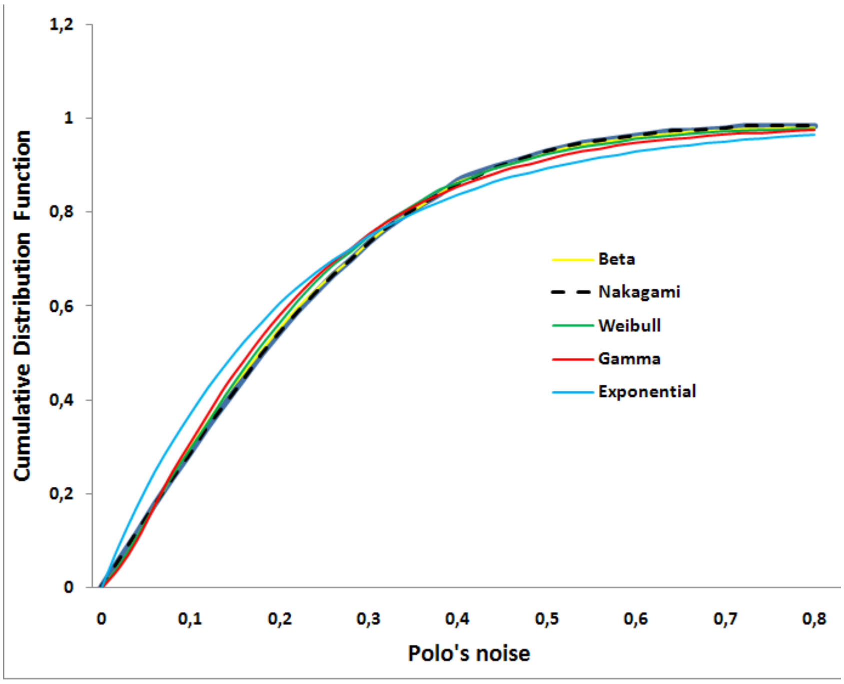
Table 4: Recognition percentage according to the KSI parameter and KL distances between measured and generated laws for Polo's model.

	class C_1		*class C_2*		*class C_3*	
	Beta	*Nakagami*	*Beta*	*Nakagami*	*Gamma*	*Weibull*
KSI	*33%*	***67%***	*37,2%*	***68,2%***	*48%*	***52%***
KL	*33,2%*	***66,8%***	*38,2%*	***61,8%***	*41,2%*	***59,8%***

Results in Table 4 confirm that the *IC*-based method allows the identification of the probabilistic distributions introduced by the Polo's model in case of low (C_3) or high cloudiness (C_1). In the case of partial cloudiness (C_2), the validation process recognizes the Nakagami law as being the distribution law of measured data, whereas the proximity of the IC values does not permit us to definitively decide between Nakagami and Beta in the identification process. In Figure 1, we provide the *pdf* (a) and the *cdf* (b) of five candidate laws in the event of the Polo’s model to the disputed case of class C_2 for 10000 values. We see the similarity of the curves, and their behavior is close to the measured values for the Beta and Nakagami laws, and thus it is difficult to decide definitively between these two laws in case of partial cloudiness. Despite this difficulty, the results are converging since the *IC*-based method clearly identifies two probabilistic distributions for modeling the random term of the Polo's model, and the validation process recognizes one of these two probabilistic distributions.



(a) *Probability density functions*



(b) *Cumulative density functions*

Figure 1: The pdfs (a) and cdfs (b) of data generated by the Polo's model in the case of C_2 (partly cloudy sky) and measured solar irradiance data

6. Conclusion

In this article, we have tried to improve modeling of the distributions involved in generation models of synthetic solar irradiance in the inter-tropical zone.

We analyzed 14 years of solar ground irradiance measurements from two weather stations located in French Guiana with a method based on *IC*. This method has permitted the identification of the probability laws that best describe the statistical distribution of each random term playing in two generation models of synthetic solar irradiance: Aguiar and Pereira's *TAG* model and Polo's model.

The identified probability laws were validated by comparing the synthetic data generated over 11 different years with in situ measured data and by using the *KL* divergence and *KSI* parameter as comparison criteria. A strong correlation was noted in Aguiar and Pereira's *TAG* model between the identified and validated probabilistic laws. This result demonstrates the non-Gaussian nature of the random term of the original autoregressive model (1). Polo's model also found good matches between these two probability laws in cases of low and high cloudiness.

In the case of partial cloudiness, the proximity of the values of identification criteria does not permit us to definitively decide between these two laws, whereas the validation process recognizes the Nakagami law as being the distribution law of measured data.

In conclusion, a new *IC*-based method has been defined and implemented on two models with results that ensure the identification of the probability laws that best describe the statistical distribution of the random terms of the two models. This method permits the modeling of synthetic solar irradiance data comparable in their statistical content to the measured solar irradiance data. This method could be extended to other measurement sites and applied to other synthetic generation models of hourly or daily solar data to validate these conclusions on a larger scale.

Acknowledgments

The authors thank the FEDER European program in French Guiana and Meteo France, who enabled the realization of this study within the SOLAREST research project.

Appendix

We recall here the probability distribution of the various candidate laws used in this paper.

1. For the *TAG* model, three laws are considered:

• The Gaussian law

$$f(x,\mu,o) = \frac{1}{\sqrt{2\pi o}} e^{-\frac{(x-\mu)^2}{2o^2}}, \quad x \in R$$

With μ being the average of the distribution and σ the standard deviation of distribution.

- The extreme value law

$$f(x,\mu,\sigma) = \sigma^{-1} e^{\frac{(x-\mu)}{\sigma}} e^{-e^{\frac{(x-\mu)}{\sigma}}}, \quad x \in R$$

withμ and σ being the parameters of the form of the distribution.

- The logistic law

$$f(x,\mu,s) = \frac{1}{s\left(1+e^{\frac{x-\mu}{s}}\right)^2} e^{-\frac{(x-\mu)}{s}}, \quad x \in R$$

With μ being the mean and s a parameter of the form linked to the variance.

2. For the Polo-based model, nine laws are considered:

- The Beta law

$$f(x,\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 y^{\alpha-1}(1-y)^{\beta-1}dy} \quad x \in [0,1]$$

= 0 otherwise

with both α and β being the form and distribution parameters.

- The Gamma law

$$f(x,\alpha,\beta) = \frac{1}{\Gamma(\alpha)} x^{\alpha-1} \beta^{\alpha} e^{-\beta x} \quad x \geq 0$$

= 0 otherwise

With α being a parameter of form and distribution,β an intensity parameter, and $\Gamma(\alpha)$ the gamma function.

- The exponential law

$$f(x,\mu) = \frac{1}{\mu} e^{-\frac{x}{\mu}}, \quad x \geq 0$$

$= 0$ otherwise

With μ being the mean and the distribution.

- The Nakagami law

$$f(x,m,\omega) = 2\left(\frac{m}{\omega}\right)^m \frac{1}{\Gamma(m)} x^{2m-1} e^{-\frac{m}{\omega}x^2}, \quad x > 0$$

$= 0$ otherwise,

With m being a form parameter, ω a parameter permitting to control the propagation of the distribution, and $\Gamma(m)$ the gamma function.

- The Rayleigh law

$$f(x,m) = \frac{x}{m^2} e^{-\frac{x^2}{2m^2}} \quad x \geq 0$$

$= 0$ otherwise,

With m being the distribution mode.

- The Rice law

$$f(x,s,\sigma) = I_0\left(\frac{xs}{\sigma^2}\right)^m \frac{x}{\sigma^2} e^{-\left(\frac{x^2+s^2}{2\sigma^2}\right)}, \quad x \geq 0$$

$= 0$ otherwise,

With s and σ being the form and distribution parameters, andI_0 being the Bessel function of the 1^{st} kind of the 0^{th} order.

- The Weibull law with two parameters

$$f(x,s,\sigma) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}, \quad x \geq 0$$

$= 0$ otherwise,

With k being a form parameter and λ a scale parameter.

- The inverse Gaussian law

$$f(x,\mu,\lambda) = \sqrt{\frac{\lambda}{2\pi x^3}} e^{-\frac{\lambda(x-\mu)^2}{2\mu^2 x}}, \quad x > 0$$

$= 0$ otherwise,

With μ being the mean and λ a form parameter.

- The normal-log law

$$f(x,\mu,\sigma)=\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}, \quad x>0$$

= 0 otherwise,

With μ being the mean and σ the standard deviation of the variable logarithm.

List of abbreviation

Clearness index definition

k_t : clearness index

I_{oh} : global irradiance on a horizontal plane

I_{sc} : irradiance produced by the solar constant

E_0 : correction factor of eccentricity

θ_z : solar zenithal angle

TAG model

$k_t(h)$: clearness index of the *TAG* model

$y(h)$: normalized clearness index of the *TAG* model

$r_{TAG}(h)$: random term of the *TAG* model

$\phi(K_T)$: correlation coefficient depending on the K_T index

h : hourly variable time

K_T : monthly average of the daily clearness index

$k_{tm}(K_T,h)$: hourly average of $k_t(h)$

$\sigma(K_T, h)$: standard deviation of $k_t(h)$

Polo model

$k_t(h)$: synthetic clearness index of the Polo model at a time h

$k_{tm}(j)$: average daily value of the clearness index for day j

$A(h)$: random amplitude of the fluctuation of the clearness index for the hour h

sign(s) : sign of the random signal

s : realization of a normal Gaussian distribution centered with zero mean and standard deviation unit

h : time

References

Aguiar, R.J., Collares-Pereira, M., Conde, J.P., 1988. Simple procedure for generating sequences of daily radiation values using a library of Markov transition matrices. Solar Energy. 40(3), 269-279.

Aguiar, R., Collares-Pereira, M., 1992. TAG: A time-dependent, autoregressive, Gaussian model for generating synthetic hourly radiation. Solar Energy. 49(3), 167-174.

Akaike, H., 1974. A new look at the statistical model identification. IEEE Trans. on Automatic Control. 19(6),716-723.

AlataO., Olivier C., Pousset Y.,2013. Law recognitions by information criteria for the statistical modeling of small scale fading of the radio mobile channel. Signal Processing. 93(5), 1064-1078.

Amado, M., Poggi, F., 2012. Towards solar urban planning: A new step for better energy performance. Energy Procedia. 30, 1261-1273.

Bilbao, J., Miguel, A., Franco J.A., Ayuso, A., 2004. Test reference year generation and evaluation methods in the continental Mediterranean area. Journal of Applied Meteorology. 43, 390-400.

Boland, J.,1995..Time-series analysis of climatic variables. Solar Energy. 55(5), 377-388.

Boland. J., 2008. Time series modelling of solar radiation, in: Badescu, V.(Ed.),Modeling Solar Radiation at the Earth's Surface: Recent Advances. Springer-Verlag, Berlin, pp. 283-312.

El Matouat, A., Hallin, M., 1996. Order selection, stochastic complexity and Kullback–Leibler information, in Athens Conference on Applied Probability and Time Series Analysis: Volume II: Time Series Analysis In Memory of E.J. Hannan. Springer-Verlag, Berlin, pp 291-299.

Espinar, B., Ramirez, L., Drews, A., Beyer, H.G., Zarzalejo, L.F., Polo, J., Martin, L., 2009. Analysis of different comparison parameters applied to solar radiation data from satellite and German radiometric stations. Solar Energy. 83, 118–125.

Graham, V.A., Hollands, K.G.T., Unny, T.E., 1988. A time series model for K_t with application to global synthetic weather generation. Solar Energy. 40(2), 83-92.

Hannan, E.J., Quinn, B.G., 1979. The determination of the order of an autoregression. . Journal of the Royal Statistical Society. 41(2), 190-195.

Hansen, C.W., Stein, J.S., Ellis, A., 2010. Statistical criteria for characterizing irradiance time series. Sandia Report, SAND2010-7314.

Jouzel, F., Olivier, C., El Matouat, A., 1998. Information criteria based edge detection, in: Theodoridis, S. (Ed.), Signal Processing IX: Proceedings of EUSIPCO-98, Ninth European Signal Processing Conference, Rhodes (Greece), 8-11 September 1998.Typorama Editions, pp. 997-1000.

- Larrañeta, M., Moreno-Tejera, S., Silva-Pérez, M.A., Lillo-Bravo, I. 2015. An improved model for the synthetic generation of high temporal resolution direct normal irradiation time series. Solar Energy. 122, 517-528

Linares-Rodríguez, A., Antonio Ruiz-Arias, J.,Pozo-Vázquez, D.J., Tovar-Pescador.,2011. Generation of synthetic daily global solar radiation data based on ERA-Interim reanalysis and artificial neural networks. Energy. 36(8), 5356-5365.

Marie-Joseph, I., Linguet, L., Gobinddass, M.L., Wald, L., 2013. On the applicability of the Heliosat-2 method to assess surface solar irradiance in the Intertropical Convergence Zone, French Guyana. International Journal of .Remote Sensing. 34(8), 3012-3027.

Mellit, A., Kalogirou, S.A., Shaari, S., Salhi, H., Hadj Arab, A., 2008. Methodology for predicting sequences of mean monthly clearness index and daily solar radiation data in remote areas: Application for sizing a stand-alone PV system. Renewable Energy. 33(7), 1570-1590.

Muselli, M., Poggi, P., Notton, G., Louche, A., 1998. Improved procedure for stand-alone photovoltaic systems sizing using METEOSTAT satellite images. Solar Energy. 62, 429-444.

Olivier, C., Alata, O., 2009. The information criteria: Examples of applications in image and signal processing, in Optimization in Image and Signal Processing. P. Siarry ed., ITSE Ltd., London and John Wiley& Sons, Inc., NJ, pp. 79-110.

Poggi, P., Notton, G., Muselli, M., Louche, A., 2000. Stochastic study of hourly total solar radiation in Corsica using a Markov model. International Journal of Climatology 20, 1843–1860.

Polo, J., Zarzalejo, L.F., Marchante, R., Navarro, A.A., 2011. A simple approach to the synthetic generation of solar irradiance time series with high temporal resolution. Solar Energy. 85, 1164-1170.

Rissanen, J., 1989. Stochastic Complexity in Statistical Inquiry. World Scientific, New Jersey.

Schwarz, G., 1978. Estimating the dimension of a model. Annals of Statistics. 6, 461-464.

Stanhill, G., Cohen, S., 2001. Global dimming: A review of the evidence for a widespread and significant reduction in global radiation with discussion of its probable causes and possible agricultural consequences. Agricultural and Forest Meteorology. 107(4), 255-278.

Tiba, C., Fraidenraich, N., 2004. Analysis of monthly time series of solar radiation and sunshine hours in tropical climates. Renewable Energy. 29(7),1147-1160.

WMO, 2008. Guide to Meteorological Instruments and Methods of Observation, seventh ed., World Meteorological Organization, Switzerland.